Solutions Manual

Power Electronics

CIRCUITS, DEVICES, AND APPLICATIONS

THIRD EDITION

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CHAPTER 2

POWER SEMICONDUCTOR DIODES AND CIRCUITS

$$\begin{array}{l} \underline{Problem \ 2-1} \\ t_{rr} = 5 \ \mu s \ and \ di/dt = 80 \ A/\mu s \\ (a) \ From Eq. \ (2-10), \\ Q_{RR} = 0.5 \ (di/dt \) \ t_{rr}^2 \ = 0.5 \ x \ 80 \ x \ 5^2 \ x \ 10^{-6} = 1000 \ \mu C \\ (b) \ From Eq. \ (2-11), \end{array}$$

$$I_{RR} = \sqrt{2Q_{RR}} \frac{di}{dt} = \sqrt{2 \times 1000 \times 80} = 400 A$$

Problem 2-2

 V_T = 25.8 mV, V_{D1} = 1.0 V at I_{D1} = 50 A, and V_{D2} = 1.5 V at I_{D2} = 600 A Taking natural (base e) logarithm on both sides of Eq. (2-3),

$$\ln I_D = \ln I_S + \frac{v_D}{\eta V_T}$$

which, after simplification, gives the diode voltage V_D as

$$v_D = \eta V_T \ In\left(\frac{I_D}{I_S}\right)$$

If I_{D1} is the diode current corresponding to diode voltage $V_{\text{D1}},$ we get

$$V_{D1} = \eta V_T \ In \left(\frac{I_{D1}}{I_S}\right)$$

Similarly, if V_{D2} is the diode voltage corresponding to the diode current I_{D2} , we get

$$V_{D2} = \eta V_T \ In\left(\frac{I_{D2}}{I_S}\right)$$

Therefore, the difference in diode voltages can be expressed by

$$V_{D2} - V_{D1} = \eta V_T \ In \left(\frac{I_{D2}}{I_S}\right) - \eta V_T \ In \left(\frac{I_{D1}}{I_S}\right) = \eta V_T \ In \left(\frac{I_{D2}}{I_{D1}}\right)$$
(a) For V_{D2} = 1.5 V, V_{D1} = 1.0 V, I_{D2} = 600 A, and I_{D1} = 50 A,
1.5-1.0 = $\eta \times 0.0258 \times In \left(\frac{600}{50}\right)$, which give $\eta = 7.799$
(b) For V_{D1} = 1.0 V, I_{D1} = 50 A, and $\eta = 7.999$

$$1.0 = 7.799 \times 0.0258 In \left(\frac{50}{I_S}\right)$$
, which gives $I_s = 0.347 A$.

Problem 2-3

$$\begin{split} V_{D1} &= V_{D2} = 2000 \text{ V}, \text{ } \text{R}_1 = 100 \text{ k}\Omega \\ \text{(a) From Fig. P2-3, the leakage current are: } I_{S1} = 17 \text{ mA and } I_{S2} = 25 \text{ mA} \\ I_{R1} &= V_{D1}/R_1 = 2000/100000 = 20 \text{ mA} \\ \text{(b) From Eq. (2-12), } I_{S1} + I_{R1} = I_{S2} + I_{R2} \\ \text{or } 17 + 20 = 25 + I_{R2}, \text{ or } I_{R2} = 12 \text{ mA} \\ \text{R}_2 &= 2000/12 \text{ mA} = 166.67 \text{ k}\Omega \end{split}$$

 $\frac{\text{Problem 2-4}}{\text{For V}_{\text{D}} = 1.5 \text{ V}, \text{ Fig. P2-3 gives I}_{\text{D1}} = 140 \text{ A and I}_{\text{D2}} = 50 \text{ A}$

 $\begin{array}{l} \underline{Problem \ 2-5} \\ I_T = 200 \ \text{A}, \ v = 2.5 \\ I_1 = I_2, \ I_1 = I_T/2 = 200/2 = 100 \ \text{A} \\ \hline For \ I_1 = 100 \ \text{A}, \ Fig. \ P2-3 \ yields \ V_{D1} = 1.1 \ \text{V} \ and \ V_{D2} = 1.95 \ \text{V} \\ v = V_{D1} + I_1 \ \text{R}_1 \ 0r \ 2.5 = 1.1 + 100 \ \text{R}_1 \ or \ \text{R}_1 = 14 \ m\Omega \\ v = V_{D2} + I_2 \ \text{R}_2 \ 0r \ 2.5 = 1.95 + 100 \ \text{R}_2 \ or \ \text{R}_2 = 5.5 \ m\Omega \end{array}$

Problem 2-6

$$\begin{split} &R_1 = R_2 = 10 \ k\Omega, \ V_s = 5 \ kV, \ I_{s1} = 25 \ mA, \ I_{s2} = 40 \ mA \\ &From Eq. \ (2-12), \ I_{S1} + I_{R1} = I_{S2} + I_{R2} \\ &or \ I_{S1} + V_{D1} \ /R_1 = I_{S2} + V_{D2} \ /R_2 \\ &25 \ x \ 10^{-3} + V_{D1} \ /10000 = 40 \ x \ 10^{-3} + V_{D2} \ /10000 \\ &V_{D1} + V_{D2} = V_s = 5000 \\ &Solving \ for \ V_{D1} \ and \ V_{D2} \ gives \ V_{D1} = 2575 \ V \ and \ V_{D2} = 2425 \ V \end{split}$$

Problem 2-7

 $t_1 = 100 \ \mu s, t_2 = 300 \ \mu s, t_3 = 500 \ \mu s, f = 250 \ Hz, f_s = 250 \ Hz, I_m = 500 \ A$ and $I_a = 200 \ A$ (a) The average current is $I_{av} = 2I_m \ ft_1/\pi - I_a \ (t_3 - t_2)f = 7.96 - 10 = -2.04$ A.

(b) For sine wave, $I_{r1} = I_m \sqrt{ft_1/2} = 55.9$ A and for a rectangular negative wave, $I_{r2} = I_a \sqrt{f(t_3 - t_2)} = 44.72$ A

The rms current is $I_{rms} = \sqrt{55.92^2 + 44.722^2} = 71.59 \text{ A}$

(c) The peak current varies from 500 A to -200 A.

Problem 2-8

t₁ = 100 µs, t₂ = 200 µs, t₃ = 400 µs, t₄ = 800 µs, f = 250 Hz, I_a = 150 A, I_b = 100 A and I_p = 300 A (a) The average current is I_{av} = I_a ft₃ + I_b f(t₅ - t₄) + 2(I_p - I_a) f(t₂ - t₁)/π = 15 + 5 + 2.387 = 22.387 A. (b) $I_{r1} = (I_p - I_a) \sqrt{f(t_2 - t_1)/2} = 16.77 \text{ A},$ $I_{r2} = I_a \sqrt{ft_3} = 47.43 \text{ A} \text{ and } I_{r3} = I_b \sqrt{f(t_5 - t_4)} = 22.36 \text{ A}$ The rms current is $I_{rms} = \sqrt{(16.772^2 + 47.432^2 + 22.362^2)} = 55.05 \text{ A}$

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<u>Problem 2-9</u> R = 22 Ω , C = 10 μ F, V_o = 220 V $0 = v_R + v_C = v_R + \frac{1}{C} \int i dt + v_C (t = 0)$

With initial condition: $v_c(t=0) = -V_o$, the current is

$$i(t) = \frac{V_o}{R}e^{-t/RC}$$

The capacitor voltage is

$$v_{C}(t) = -Ri = -V_{o}e^{-t/RC} = -220e^{-t \times 10^{6}/220}$$

(b) The energy dissipated is

$$W = 0.5 C V_0^2 = 0.5 \times 10 \times 10-6 \times 220 \times 220 = 0.242 J.$$

Problem 2-10

 $R = 10 \Omega$, L = 5 mH, Vs = 220 V, $I_1 = 10 A$

The switch current is described by

$$V_{S} = L\frac{di}{dt} + Ri$$

With initial condition: $i(t=0) = I_1$,

$$i(t) = \frac{V_s}{R}(1 - e^{-tR/L}) + I_1 e^{-tR/L} = 22 - 12e^{-2000t} A$$

Problem 2-11

$$V_S = L\frac{di}{dt} + \frac{1}{C}\int i\,dt + v_C(t=0)$$

With initial condition: $i(t=0) = I_0$ and $v_c(t=0) = 0$, we get

$$i(t) = I_o \cos(\omega_o t) + V_S \sqrt{\frac{C}{L}} \sin(\omega_o t)$$

The capacitor voltage is

$$v_{c}(t) = \frac{1}{C} \int i \, dt = I_{o} \sqrt{\frac{L}{C}} \sin(\omega_{o}t) - V_{S} \cos(\omega_{o}t) + V_{S}$$

where $\omega_{o} = 1/\sqrt{CL}$

Problem 2-12

Fig. p2-12a:
(a) L di/dt or
$$i(t) = V_s t/L$$

(b) di/dt = V_s/L;

(d) di/dt (at t= 0) = V_s/L .

Fig. p2-12b:

(a)
$$\frac{1}{C} \int i dt + R i = V_S - V_o \text{ or } i(t) = \frac{V_S - V_o}{R} e^{-t/RC}$$

(b)
$$\frac{di}{dt} = \frac{v_s - v_o}{R^2 C} e^{-t/RC}$$

(d) At t = 0, di/dt =
$$(V_s - V_o)/(R^2 C)$$

Fig. p2-12c:

(a)
$$L\frac{di}{dt} + Ri = V_S \text{ or } i(t) = \frac{V_S}{R}e^{-tR/L}$$

(b)
$$\frac{di}{dt} = -\frac{V_s}{L}e^{-tR/L}$$

(d) At
$$t = 0$$
, $di/dt = V_s/L$

Fig. p2-12d:

(a)
$$V_S = L\frac{di}{dt} + \frac{1}{C}\int i dt + v_C(t=0)$$

With initial condition: i(t=0) = 0 and $v_c(t=0) = V_o$,

$$\dot{i}(t) = \left(V_S - V_o\right) \sqrt{\frac{C}{L}} \sin(\omega_o t) = I_p \sin(\omega_o t)$$

where $\omega_o = 1/\sqrt{LC}$

(b)
$$\frac{di}{dt} = \frac{V_S - V_O}{L} \cos(\omega_O t)$$

(d) At
$$t = 0$$
, $di/dt = (V_s - V_o)/L$

Fig. p2-12e:

At t=0, the inductor behaves as an open circuit and a capacitor behaves as a short circuit. Inductor L₁ limits the initial di/dt only. Thus, the initial di/dt is $di/dt = V_s/L_1 = V_s/20 \mu H = V_s/20 A/\mu s$

Problem 2-13 V_s = 220 V, L = 5 mH, C = 10 μF, R = 22 Ω and V_o = 50 V (a) From Eq. (2-40), α = 22 x 10³/(2 x 5) = 2200 From Eq. (2-41), ω_o = 1/√(LC) = 4472 rad/s $\omega_r = \sqrt{4472^2 \cdot 2200^2}$ = 3893 rad/s

Since $\alpha < \omega_o$, it is an under-damped case and the solution is of the for

$$i(t) = e^{-\alpha t} [A_1 \cos(\omega_r t) + A_2 \sin(\omega_r t)]$$

At t= 0, i(t=0) = 0 and this gives $A_1 = 0$.

$$i(t) = e^{-\alpha t} A_2 \sin(\omega_r t)$$

$$\frac{di}{dt} = \omega_r \cos(\omega_r t) A_2 e^{-\alpha t} - \alpha \sin(\omega_r t) A_2 e^{-\alpha t}$$
$$\frac{di}{dt}\Big|_{t=0} = \omega_r A_2 = \frac{V_s}{L}$$

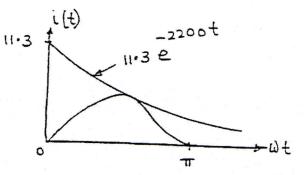
or $A_2 = V_s/(\omega_r L) = 220 \times 1000/(3893 \times 5) = 11.3$ The final expression for current i(t) is

$$i(t) = 11.3 \times \sin(3893t) e^{-2200t}$$
 A

(b) The conduction time is

 $\omega_r t_1 = \pi$ or $t_1 = \pi/3893 = 807 \ \mu s$

(c) The sketch for i(t) is shown.



Problem 2-14

- $V_s = 200 \text{ V}, L_m = 150 \text{ }\mu\text{H}, N_1 = 10, N_2 = 200 \text{ and } t_1 = 100 \text{ }\mu\text{s}$ The turns ratio is a = $N_2/N_1 = 200/10 = 20$
- (a) From Eq. (2-52) the reverse voltage of diode,

 $v_D = 200 \times (1 + 20) = 4620 V$

(b) From Eq. (2-55) the peak value of primary current,

 $I_o = 220 \times 100/150 = 146.7 \text{ A}$

(c) The peak value of secondary current Io' = Io/a = 146.7/20 = 7.3 A

(d) From Eq. (2-58) the conduction time of diode,

 $t2 = 20 \times 100 = 2000 \ \mu s.$

(e) The energy supplied by the source

$$W = \int_0^1 v \, i \, dt = \int_0^1 V_S \frac{V_S}{L_m} t \, dt = \frac{1}{2} \frac{V_S^2}{L_m} t_1^2$$

From Eq. (2-55), W = 0.5 $L_m I_o^2$ = 0.5 x 150 x 10-6 x 146.72 = 1.614 J

Problem 2-15 (a) $i_c = i_d + I_m$ $v_c = \frac{1}{C} \int i_c dt + v_c (t = 0) = -L \frac{di_d}{dt} = -L \frac{di_c}{dt}$ With initial condition: $i_c t=0$) = I_m and $v_c(t=0) = -V_s$, $i(t) = V_S \sqrt{\frac{C}{L}} \sin(\omega_o t) + I_m \cos(\omega_o t)$ where $\omega_o = 1/\sqrt{LC}$ $v_c(t) = \frac{1}{C} \int i_c(t) dt = I_m \sqrt{\frac{L}{C}} \sin(\omega_o t) - V_S \cos(\omega_o t)$ $i_d(t) = V_S \sqrt{\frac{C}{L}} \sin(\omega_o t) + I_m \cos(\omega_o t) - I_m$ (b) For $i_d(t = t_1) = 0$ $i_d(t = t_1) = V_S \sqrt{\frac{C}{L}} \sin(\omega_o t_1) + I_m \cos(\omega_o t_1) - I_m = 0$ or $\cos(\alpha) \sin(\omega_o t_1) + \sin(\alpha) \cos(\omega_o t_1) = \frac{1}{\sqrt{1 + x^2}}$

or
$$\sin(\omega_o t_1 + \alpha) = \frac{1}{\sqrt{1 + x^2}}$$

which gives the time t_1

$$\omega_{O} t_{1} = \sin^{-1} \left[\frac{1}{\sqrt{1 + x^{2}}} \right] - \alpha = \sin^{-1} \left[\frac{1}{\sqrt{1 + x^{2}}} \right] - \tan^{-1} \left(\frac{1}{x} \right)$$
where $x = \frac{V_{S}}{I_{m}} \sqrt{\frac{C}{L}}$
(c) For $v_{c}(t = t_{a}) = 0$

$$v_c(t) = I_m \sqrt{\frac{L}{C}} \sin(\omega_o t) - V_s \cos(\omega_o t) = 0$$

or $t_q = \sqrt{LC} \tan^{-1}(x)$

(d) The time for the capacitor to recharge to the supply voltage at a constant current of $\mathrm{I}_{\mathrm{m}},$ is

$$v_C(t=t_1) = V_s = \frac{1}{C} \int_0^t I_m dt$$

$$t_1 = V_s C / I_m$$

The total time for discharge and recharge is t_{2} = t_{1} + t_{q}

CHAPTER 3

DIODE RECTIFIERS

Problem 3-1

 $V_{\rm m} = 170 \text{ V}, \text{ R} = 10 \Omega, \text{ f} = 60 \text{ Hz}$

From Eq. (3-21), $V_d = 0.6366 V_m = 0.6366 \times 170 = 113.32 V$

Problem 3-2

 $V_m = 170 \text{ V}, \text{ R} = 10 \Omega, \text{ f} = 60 \text{ Hz} \text{ and } L_c = 0.5 \text{ mH}$ From Eq. (3-21), $V_{dc} = 0.6366 \text{ V}_m = 0.6366 \times 170 = 113.32 \text{ V}$ $I_{dc} = V_{dc}/\text{R} = 113.32/10 = 11.332 \text{ A}$ Since there are two commutations per cycle, Eq. (3-79) gives the output voltage reduction, $V_x = 2 \times 60 \times 0.5 \times 10^{-3} \times 11.332 = 0.679 \text{ V}$ and the effective output voltage is (113.32 - 0.679) = 112.64 V

Problem 3-3

R = 10 Ω , V_m = 170 V, f = 60 Hz For a six-phase star rectifier q = 6 in Eqs. (3-32) and from Eq. (3-32), V_{dc} = 170 (6/ π) sin (π /6) = 162.34 V

Problem 3-4

R = 10 Ω , V_m = 170 V, f = 60 Hz, L_c = 0.5 mH For a six-phase star-rectifier, q = 6 in Eq. (3-69). From Eq. (3-32), V_{dc} = 170 (6/ π) sin (π /6) = 162.34, I_{dc} = 162.34/10 = 16.234 A. Since there are six commutations per cycle, Eq. (3-79) gives the output voltage reduction, V_x = 6 x 60 x 0.5 x 10⁻³ x 16.234 = 2.92 V and the effective output voltage is (162.34 - 2.92) = 159.42 V

Problem 3-5

R = 100 Ω , V_s = 280 V, f = 60 Hz V_m = 280 x $\sqrt{2}/\sqrt{3}$ = 228.6 V From Eq. (3-40), V_{dc} = 1.6542 x 228.6 = 378.15 V

Problem 3-6

$$\begin{split} &\mathsf{R} = 100 \; \Omega, \, \mathsf{V_s} = 280 \; \mathsf{V}, \, \mathsf{f} = 60 \; \mathsf{Hz} \; \mathsf{and} \; \mathsf{L_c} = 0.5 \; \mathsf{mH} \\ &\mathsf{V_m} = 280 \; x \; \sqrt{2}/\sqrt{3} = 228.6 \; \mathsf{V} \\ &\mathsf{From} \; \mathsf{Eq.} \; (3\text{-}40), \, \mathsf{V_{dc}} = 1.6542 \; x \; 228.6 = 378.15 \; \mathsf{V} \\ &\mathsf{I_{dc}} = \; \mathsf{V_{dc}}/\mathsf{R} = 378.15/100 = 37.815 \; \mathsf{A} \\ &\mathsf{Since} \; \mathsf{there} \; \mathsf{are} \; \mathsf{six} \; \mathsf{commutations} \; \mathsf{per} \; \mathsf{cycle}, \; \mathsf{Eq.} \; (3\text{-}79) \; \mathsf{gives} \; \mathsf{the} \; \mathsf{output} \\ &\mathsf{voltage} \; \mathsf{reduction}, \; \mathsf{V_x} = 6 \; \mathsf{x} \; 60 \; \mathsf{x} \; 0.5 \; \mathsf{x} \; 10^{-3} \; \mathsf{x} \; 37.815 = 6.81 \; \mathsf{V} \; \mathsf{and} \; \mathsf{the} \\ &\mathsf{effective} \; \mathsf{output} \; \mathsf{voltage} \; \mathsf{is} \; (378.15 - 6.81) = 371.34 \; \mathsf{V} \end{split}$$

Problem 3-7

 $P_{dc} = (0.6366 V_m)^2/R$ and $P_{ac} = V_s I_s = V_m^2/2R$ TUF = $P_{dc}/P_{ac} = 0.6366^2 \times 2 = 0.8105$ and the de-rating factor of the transformer is 1/TUF = 1.2338.

Problem 3-8

Problem 3-9

 $V_m = 170 \text{ V}, \text{ f} = 60 \text{ Hz}, \text{ R} = 15 \Omega \text{ and } \omega = 2\pi \text{f} = 377 \text{ rad/s}$ From Eq. (3-22), the output voltage is

$$w_L(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos(2\omega t) - \frac{4V_m}{15\pi} \cos(4\omega t) - \frac{4V_m}{35\pi} \cos(6\omega t) - ..\infty$$

The load impedance, $Z = R + j(n\omega L) = \sqrt{R^2 + (n\omega L)^2} \angle \theta_n$

and $\theta_n = \tan^{-1}(n\omega L/R)$

and the load current is given by

$$i_{L}(t) = I_{dc} - \frac{4V_{m}}{\pi\sqrt{R^{2} + (n\omega L)^{2}}} \left[\frac{1}{3}\cos(2\omega t - \theta_{2}) - \frac{1}{15}\cos(4\omega t - \theta_{4}) - \frac{1}{35}\cos(6\omega t - \theta_{6}) - ...\infty \right]$$

where

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

The rms value of the ripple current is

$$I_{ac}^{2} = \frac{(4V_{m})^{2}}{2\pi^{2}[R^{2} + (2\omega L)^{2}]} \left(\frac{1}{3}\right)^{2} + \frac{(4V_{m})^{2}}{2\pi^{2}[R^{2} + (4\omega L)^{2}]} \left(\frac{1}{15}\right)^{2} + \frac{(4V_{m})^{2}}{2\pi^{2}[R^{2} + (6\omega L)^{2}]} \left(\frac{1}{35}\right)^{2} + ...\infty$$

Considering only the lowest order harmonic (n = 2) and neglecting others,

$$I_{ac} = \frac{4V_m}{\sqrt{2}\pi\sqrt{R^2 + (2\omega L)^2}} \left(\frac{1}{3}\right)$$

Using the value of I_{dc} and after simplification, the ripple factor is

$$RF = \frac{I_{ac}}{I_{dc}} = \frac{0.481}{\sqrt{1 + (2\omega L/R)^2}} = 0.04$$
$$0.481^2 = 0.04^2 [1 + (2 \times 377 \text{ L}/15)^2] \text{ or } \text{L} = 238.4 \text{ mH}$$

Problem 3-10

 $V_m = 170 V$, f = 60 Hz, R = 15 Ω and $\omega = 2\pi f = 377 rad/s$ For q = 6, Eq. (3-39) gives the output voltage as

$$v_L(t) = 0.9549 V_m \left[1 + \frac{2}{35} \cos(6\omega t) - \frac{2}{143} \cos(12\omega t) + ..\infty \right]$$

The load impedance, $Z = R + j(n\omega L) = \sqrt{R^2 + (n\omega L)^2} \angle \theta_n$

and $\theta_n = \tan^{-1}(n\omega L/R)$

and the load current is

$$i_{L}(t) = I_{dc} - \frac{0.9549V_{m}}{\sqrt{R^{2} + (n\omega L)^{2}}} \left[\frac{2}{35} \cos(6\omega t - \theta_{6}) - \frac{2}{143} \cos(12\omega t - \theta_{12}) + ..\infty \right]$$

where

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.9549V_m}{R}$$

The rms value of the ripple current is

$$I_{ac}^{2} = \frac{(0.9549V_{m})^{2}}{2[R^{2} + (6\omega L)^{2}]} \left(\frac{2}{35}\right)^{2} + \frac{(0.9549V_{m})^{2}}{2[R^{2} + (12\omega L)^{2}]} \left(\frac{2}{143}\right)^{2} + ..\infty$$

Considering only the lowest order harmonic (n = 6) and neglecting others,

$$I_{ac} = \frac{0.9549V_m}{\sqrt{2}\sqrt{R^2 + (6\omega L)^2}} \left(\frac{2}{35}\right)$$

Using the value of I_{dc} and after simplification, the ripple factor is

$$RF = \frac{I_{ac}}{I_{dc}} = \frac{1}{\sqrt{2} \times \sqrt{1 + (6\omega L/R)^2}} \left(\frac{2}{35}\right) = 0.02$$

0.0404² = 0.02² [1 + (6 x 377 L/15)²] or L = 11.64 mH

Problem 3-11

$$\begin{split} &\mathsf{E} = 20 \;\mathsf{V}, \; I_{dc} = 10 \;\mathsf{A}, \; \mathsf{V_p} = 120 \;\mathsf{V}, \; \mathsf{V_s} = \mathsf{V_p/n} = 120/2 = 60 \;\mathsf{V} \\ &\mathsf{V_m} = \sqrt{2} \;\mathsf{V_s} = \sqrt{2} \;\mathsf{x} \; 60 = 84.85 \;\mathsf{V} \\ &(\mathsf{i}) \;\mathsf{From} \;\mathsf{Eq}. \; (3\text{-}17), \; \alpha = \sin^{-1} \;(20/84.85) = 15.15^\circ \; \text{or} \; 0.264 \; \text{rad} \\ &\beta = 180 \; \text{-} \; 15.15 = 164.85^\circ \\ &\mathsf{The} \; \mathsf{conduction} \; \mathsf{angle} \; \mathsf{is} \; \delta = \beta \; \text{-} \; \alpha = 164.85 \; \text{-} \; 15.15 = 149.7^\circ \end{split}$$

(ii) Equation (3-18) gives the resistance R as

$$R = \frac{1}{2\pi I_{dc}} \left[2V_m \cos\alpha + 2\alpha E - \pi E \right]$$

 $R = \frac{1}{2\pi \times 10} \Big[2 \times 84.85 \times \cos 15.15^{\circ} + 2 \times 20 \times 0.264 - \pi \times 20 \Big] = 1.793 \ \Omega$

(iii) Equation (3-19) gives the rms battery current I_{rms} as

$$I_{rms}^{2} = \frac{1}{2\pi R^{2}} \left[\left(\frac{V_{m}^{2}}{2} + E^{2} \right) \times (\pi - 2\alpha) + \frac{V_{m}^{2}}{2} \sin 2\alpha - 4V_{m} E \cos \alpha \right] = 272.6$$

or $I_{rms} = \sqrt{272.6} = 16.51 \text{ A}$

The power rating of R is $P_R = 16.51^2 \times 1.793 = 488.8 \text{ W}$ (iv) The power delivered P_{dc} to the battery is $P_{dc} = E I_{dc} = 20 \times 10 = 200 \text{ W}$ $h P_{dc} = 100 \text{ or } h = 200/P_{dc} = 200/200 = 1 \text{ hr}$ (v) The rectifier efficiency η is

$$\eta = \frac{P_{dc}}{P_{dc} + P_R} = \frac{200}{200 + 488.8} = 29\%$$

(vi) The peak inverse voltage PIV of the diode is $PIV = V_m + E = 84.85 + 20 = 104.85 V$

Problem 3-12

It is not known whether the load current is continuous or discontinuous. Let us assume that the load current is continuous and proceed with the solution. If the assumption is not correct, the load current will be zero current and then move to the case for a discontinuous current.

(a) $R = 5 \Omega$, L = 4.5 mH, f = 60 Hz, $\omega = 2 \pi \times 60 = 377 \text{ rad/s}$, $V_s = 120 \text{ V}$, $Z = [R^2 + (\omega \text{ L})^2]^{1/2} = 5.28 \Omega$, and $\theta = \tan^{-1}(\omega \text{ L/R}) = 18.74^{\circ}$

(i) The steady-state load current at $\omega t = 0$, $I_1 = 6.33$ A. Since $I_1 > 0$, the load current is continuous and the assumption is correct.

(ii) The numerical integration of i_L in Eq. (3-27) yields the average diode current as $I_d = 8.8$ A

(iii) By numerical integration of i_{L}^{2} between the limits $\omega t = 0$ to π , we get the rms diode current as $I_{r} = 13.83$ A.

(iv) The rms output current I_{rms} = $\sqrt{2}~I_{r}$ = $\sqrt{2}~x$ 13.83 = 19.56 A

Problem 3-13

(a) R = 5 Ω , L = 2.5 mH, f = 60 Hz, $\omega = 2 \pi \times 60 = 377 \text{ rad/s}$, V_{ab} = 208 V, Z = $[R^2 + (\omega L)^2]^{1/2} = 5.09 \Omega$, and $\theta = \tan^{-1} (\omega L/R) = 10.67^{\circ}$ (i) The steady-state load current at $\omega t = \pi/3$, I₁ = 50.6 A. (ii) The numerical integration of i_L in Eq. (3-47) yields the average diode current as I_d = 17.46 A. Since I₁ > 0, the load current is continuous. (iii) By numerical integration of i_L² between the limits $\omega t = \pi/3$ to $2\pi/3$, we get the rms diode current as I_r = 30.2 A.

(iv) The rms output current $I_{rms} = \sqrt{3} I_r = \sqrt{3} \times 30.2 = 52.31 A$

Problem 3-14

RF = 5%, $R = 200 \Omega$ and f = 60 Hz

(a) Solving for C_e in Eq. (3-62),

$$C_e = \frac{1}{4 \times 60 \times 200} \left[1 + \frac{1}{\sqrt{2} \times 0.05} \right] = 315.46 \ \mu \text{F}$$

(b) From Eq. (3-61), the average load voltage $V_{dc}\xspace$ is

 $V_{dc} = 169.7 - \frac{169.7}{4 \times 60 \times 200 \times 415.46 \times 10^{-6}} = 169.7 - 11.21 = 158.49 \text{ V}$

Problem 3-15

RF = 5%, R = 200 Ω , and f = 60 Hz

(a) For a half-wave rectifier, the frequency of output ripple voltage is the same as the supply frequency. Thus, the constant 4 in Eq. (3-62) should be changed to 2.

Solving for C_e in Eq. (3-62),

$$C_e = \frac{1}{2 \times 60 \times 200} \left[1 + \frac{1}{\sqrt{2} \times 0.05} \right] = 630.92 \ \mu\text{F}$$

(b) From Eq. (3-61), the average load voltage V_{dc} is

$$V_{dc} = 169.7 - \frac{169.7}{2 \times 60 \times 200 \times 415.46 \times 10^{-6}} = 169.7 - 22.42 = 147.28 \text{ V}$$

 ω = 2 π x 60 = 377 rad/s, V_{dc} = 48 V, V_s = 120 V, V_m = $\sqrt{2}$ x 120 = 169.7 V (a) Voltage ratio x = V_{dc}/V_m = 48/169.7 = 28.28 % $\alpha = \sin^{-1}(x) = 16.43^{\circ}$ Solving Eq. (3-70) for ß gives: $\beta = 117.43^{\circ}$ Equation (3-105) gives the current ratio $I_{dc}/Ipk = 13.425$ % Thus, $I_{pk} = I_{dc}/0.13425 = 186.22 \text{ A}$ The required value of inductance is $L_e = V_m/(\omega I_{pk}) = 169.7/(377 \times 186.22) = 2.42 \text{ mH}.$ Equation (3-106) gives the current ratio $I_{rms}/Ipk = 22.59 \%$ Thus $I_{rms} = 0.2259 \times I_{pk} = 0.2259 \times 186.22 = 42.07 \text{ A}$ (b) I_{dc} = 15 A, L_e = 6.5 mH, I_{pk} = V_m/(ω L_e) = 169.7/(377 x 6.5 mH) = 69.25 A $v = I_{dc}/I_{pk} = 15/69.25 = 21.66\%$ Using linear interpolation, we get $x = x_n - (x_{n+1} - x_n) (y_n - y)/(y_{n+1} - y_n)$ = 10 - (15 - 10) (25.5 - 21.66)/(21.5 - 25.5) = 14.8% $V_{dc} = x V_m = 0.148 X 169.7 = 25.12 V$ $\alpha = \alpha_n - (\alpha_{n+1} - \alpha_n) (y_n - y)/(y_{n+1} - y_n)$ $= 5.74 - (8.63 - 5.74) (25.5 - 21.66)/(21.5 - 25.5) = 8.51^{\circ}$ $\beta = \beta_n - (\beta_{n+1} - \beta_n) (y_n - y)/(y_{n+1} - y_n)$ $= 139.74 - (131.88 - 139.74) (25.5 - 21.66)/(21.5 - 25.5) = 132.19^{\circ}.$ $z = I_{rms}/I_{pk} = z_n - (z_{n+1} - x_n) (y_n - y)/(y_{n+1} - y_n)$ = 37.06 - (32.58 - 37.06) (25.5 - 21.66)/(21.5 - 25.5) = 32.76% Thus $I_{rms} = 0.3276 \times I_{pk} = 0.3276 \times 69.25 = 22.69 \text{ A}$

(a)
f := 60 Vdc := 48 Idc := 25
$$\omega$$
 := $2 \cdot \pi \cdot f$ ω = 376.99
Vs := 120 Vm := $\sqrt{2} \cdot Vs$
x := $\frac{Vdc}{Vm}$ α := $asin(x)$ $180 \cdot \frac{\alpha}{\pi} = 16.43$
k := $\sqrt{1 - (x)^2} + \left(\frac{2}{\pi} - \frac{\pi}{2}\right)x$ $100 \cdot k = 69.49$
Ipk := $\frac{Idc}{k}$ Ipk = 35.97
Lcr := $\frac{Vm}{\omega \cdot Ipk}$ $1000 \cdot Lcr = 12.51$ mH

$$\mathbf{k}_{\mathbf{r}} := \sqrt{\frac{1}{\pi}} \left[\int_{\alpha}^{\alpha + \pi} \left[\left(\cos(\alpha) - \cos(\phi) \right) - \mathbf{x} \cdot (\phi - \alpha) \right]^2 d\phi \right]$$

 $100 \cdot k_r = 81.91$

 $Irms := k_{r} \cdot Ipk$

$$Irms = 29.47$$

(b)

$$I_{dc} := 15 Ipk := 69.25 k := \frac{100 \cdot I_{dc}}{Ipk} k = 21.66 x_n := 60 x_n 1 := 65 \alpha_n := 36.87 \alpha_n 1 := 40.54 k_n := 23.95 k_n 1 := 15.27 k_n := 31.05 kr_n 1 := 26.58 x := x_n + \left[\frac{\left(x_{n1} - x_n\right) \cdot \left(k - k_n\right)}{k_{n1} - k_n}\right] x = 61.32 Vdc := x \cdot \frac{Vm}{100} Vdc = 104.06 \alpha := \alpha_n + \left[\frac{\left(\alpha_{n1} - \alpha_n\right) \cdot \left(k - k_n\right)}{k_{n1} - k_n}\right] \alpha = 37.84 kr := kr_n + \left[\frac{\left(kr_{n1} - kr_n\right) \cdot \left(k - k_n\right)}{k_{n1} - k_n}\right] kr = 29.87 kr_n = 29.87 kr_n = 29.87 kr_n = 10.5 kr_n = 10.5 kr_n = 10.5 kr_n = 20.87 kr_n = 20.87$$

Irms :=
$$\frac{\mathbf{kr} \cdot \mathbf{lpk}}{100}$$
 Irms = 20.69

Let t_1 and t_2 be the charging and discharging time of capacitor. For a singlephase full-wave rectifier, the period of output voltage is T/2, where T is the period of the input voltage and the supply frequency is f = 1/T. $t_1 + t_2 = T/2$. If $t_2 >> t_1$ which is normally the case, $t_2 \approx T/2$ During discharging of the capacitor, the capacitor discharges exponentially and the output (or capacitor) voltage is

$$v_o(t) = V_m e^{-t/RC}$$

where V_{m} is the peak value of supply voltage.

The peak-to-peak ripple voltage is

$$v_r = v_o(t = t_1) - v_o(t = t_2) = V_m - V_m e^{-t_2/RC} = V_m [1 - e^{-t_2/RC}]$$

Since, $e^{-x} \approx 1 - x$, $v_r = V_m (1 - 1 + t_2/RC) = V_m t_2/RC = V_m/(2fRC)$ Thus, the rms value of the output voltage harmonics is

$$V_{ac} = \frac{v_r}{2\sqrt{2}} = \frac{V_m}{4\sqrt{2} f RC}$$

Problem 3-18

R = 20 Ω , L = 5 mH, f = 60 Hz, ω = 2 π f = 377 rad/s Taking a ratio of 10:1, the value of the capacitor is given by

$$\sqrt{R^2 + (6\omega L)^2} = \frac{10}{6\omega C_{\rho}}$$

or

$$C_e = \frac{10}{6 \times 377 \sqrt{R^2 + (6 \times 377L)^2}} = 192.4 \ \mu \text{F}$$

From Eq. (3-39), the rms value of the 6th harmonic is

$$V_6 = \frac{2}{35\sqrt{2}} \times 0.9549 V_m$$

From Eq. (3-64), the rms vale of the ripple voltage is

$$V_{ac} = \frac{V_6}{(n\omega)^2 L_1 C - 1} = \frac{2}{35\sqrt{2}} \times \frac{0.9549 V_m}{(6\omega)^2 L_1 C - 1}$$

 $V_{dc} = 0.9549 V_{m}$

The ripple factor is

$$RF = \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{2}}{35} \times \frac{1}{(6\omega)^2 L_1 C - 1} = 0.05$$

or $(6\omega)^2 L_1 C - 1 = 0.808$ and $L_1 = 1.837$ mH

- (a) With q = 6, Eq. (3-39) gives the output voltage as $v_L(t) = 0.9549 V_m \left[1 + \frac{2}{35} \cos(6\omega t) - \frac{2}{143} \cos(12\omega t) + ..\infty \right]$ The load impedance, $Z = R + j(n\omega L) = \sqrt{R^2 + (n\omega L)^2} \angle \theta_n$
- and $\theta_n = \tan^{-1}(n\omega L/R)$

and the load current is

 I_{dc}

$$i_{L}(t) = I_{dc} - \frac{0.9549V_{m}}{\sqrt{R^{2} + (n\omega L)^{2}}} \left[\frac{2}{35} \cos\left(6\omega t - \theta_{6}\right) - \frac{2}{143} \cos\left(12\omega t - \theta_{12}\right) + ..\infty \right]$$

where

$$=\frac{V_{dc}}{R}=\frac{0.9549V_m}{R}$$

(b) $V_m = 170 V$, f = 60 Hz, R = 200 Ω , $\omega = 2\pi$ f = 377 rad/s

The rms value of the ripple current is

$$I_{ac}^{2} = \frac{(0.9549V_{m})^{2}}{2[R^{2} + (6\omega L)^{2}]} \left(\frac{2}{35}\right)^{2} + \frac{(0.9549V_{m})^{2}}{2[R^{2} + (12\omega L)^{2}]} \left(\frac{2}{143}\right)^{2} + ..\infty$$

Considering only the lowest order harmonic (n = 6) and neglecting others,

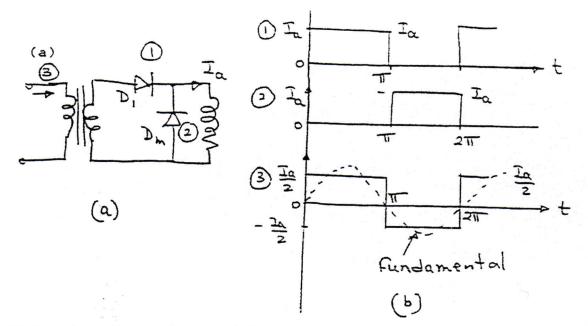
$$I_{ac} = \frac{0.9549 V_m}{\sqrt{2} \sqrt{R^2 + (6\omega L)^2}} \left(\frac{2}{35}\right)$$

Using the value of I_{dc} and after simplification, the ripple factor is

$$RF = \frac{I_{ac}}{I_{dc}} = \frac{1}{\sqrt{2} \times \sqrt{1 + (6\omega L/R)^2}} \left(\frac{2}{35}\right) = 0.02$$

 $0.0404^2 = 0.02^2 [1 + (6 \times 377 \text{ L}/200)^2] \text{ or } \text{L} = 11.64 \text{ mH}$

(a)



(b) For the primary (or supply) current, $a_o = 0$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \frac{I_a}{2} \cos(n\theta) d\theta = 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \frac{I_a}{2} \sin(n\theta) d\theta = \frac{2I_a}{n\pi}$$

$$\varphi_n = \tan^{-1} (a_n/b_n) = 0$$

$$i_s(t) = \frac{2I_a}{\pi} \left[\frac{\sin \omega t}{1} + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + ..\infty \right]$$

The rms value of the fundamental current is

$$I_1 = 2I_a/(\pi\sqrt{2})$$

The rms current is $I_s = I_a/2$. At the primary (or supply) side, PF = $I_1/I_s = 2\sqrt{2/\pi} = 0.9$ and $HF = \sqrt{(I_s/I_1)^2 - 1} = 0.4834$.

(c) For the rectifier input (or secondary) side,

$$a_{0}/2 = I_{a}/2$$

$$a_{n} = \frac{1}{\pi} \int_{0}^{\pi} I_{a} \cos(n\theta) d\theta = 0$$

$$b_{n} = \frac{1}{\pi} \int_{0}^{\pi} I_{a} \sin(n\theta) d\theta = \frac{I_{a}}{n\pi} (1 - \cos n\theta)$$

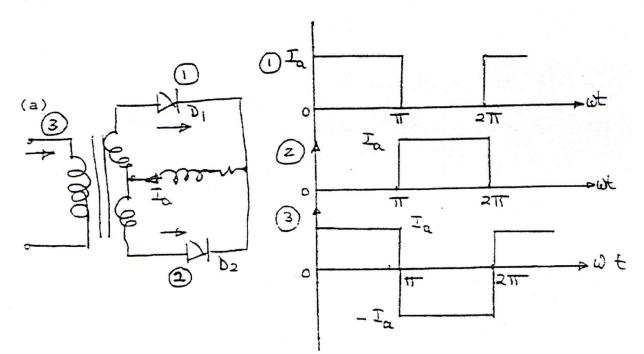
$$\phi_{n} = \tan^{-1} (a_{n}/b_{n}) = 0$$

$$C_{n} = \sqrt{(a_{n}^{2} + b_{n}^{2})} \text{ and } I_{1} = C_{1}/\sqrt{2} = \sqrt{2I_{a}/\pi}$$
and $I_{s} = I_{a}/\sqrt{2}$

$$PF = I_{1}/I_{s} = 2/\pi = 0.6366 \text{ and } HF = \sqrt{(I_{s}/I_{1})^{2} - 1} = 1.211$$

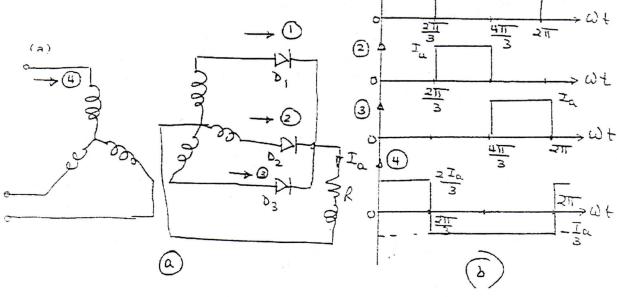
Problem 3-21

(a)



(b) For the primary (or supply) current: From Eq. (3-23), the primary current is

$$\begin{split} i_{s}(t) &= \frac{4I_{a}}{\pi} \left[\frac{\sin \omega t}{1} + \frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + ..\infty \right] \\ I_{1} &= 4I_{a}/(\pi\sqrt{2}) \\ \text{The rms current is } I_{s} &= I_{a}. \text{ PF} = I_{1}/I_{s} = 2\sqrt{2}/\pi = 0.9 \text{ and } HF = \sqrt{(I_{s}/I_{1})^{2} - 1} = 0.4834. \\ \text{(c) For the rectifier input (or secondary) current:} \\ a_{0}/2 &= I_{a}/2 \\ a_{n} &= \frac{1}{\pi} \int_{0}^{\pi} I_{a} \cos(n\theta) d\theta = 0 \\ b_{n} &= \frac{1}{\pi} \int_{0}^{\pi} I_{a} \cos(n\theta) d\theta = \frac{I_{a}}{n\pi} (1 - \cos n\theta) \\ \phi_{n} &= \tan^{-1} (a_{n}/b_{n}) = 0 \\ C_{n} &= \sqrt{(a_{n}^{2} + b_{n}^{2})} \text{ and } I_{1} = C_{1}/\sqrt{2} = \sqrt{2I_{a}/\pi} \\ \text{and } I_{s} &= I_{a}/\sqrt{2} \\ \text{PF} &= I_{1}/I_{s} = 2/\pi = 0.6366 \text{ and } HF = \sqrt{(I_{s}/I_{1})^{2} - 1} = 1.211 \\ \hline \text{Problem 3-22} \qquad \text{(a)} \end{split}$$

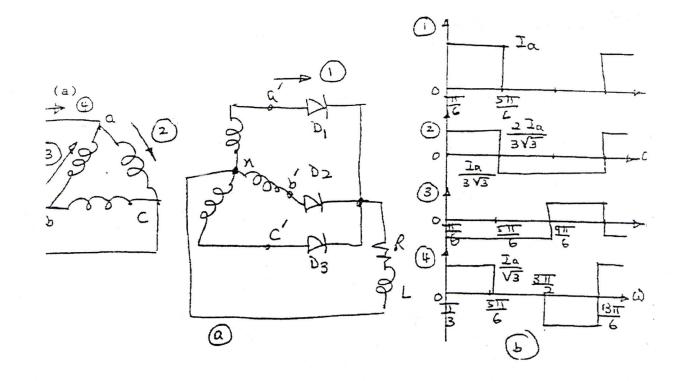


(b) For the primary (or secondary) phase (or line) current:

$$a_0/2 = 0$$

 $a_n = \frac{1}{\pi} \int_{\pi/6}^{\pi/6} \frac{2I_a}{3} \cos(n\theta) d\theta - \frac{1}{\pi} \int_{\pi/6}^{2\pi\pi/6} \frac{I_a}{3} \cos(n\theta) d\theta$
 $= \frac{2I_a}{n\pi} \cos \frac{n\pi}{2} \sin \frac{n\pi}{3}$
 $b_n = \frac{1}{\pi} \int_{\pi/6}^{\pi/6} \frac{2I_a}{3} \sin(n\theta) d\theta - \frac{1}{\pi} \int_{\pi/6}^{2\pi\pi/6} \frac{I_a}{3} \sin(n\theta) d\theta$
 $= \frac{2I_a}{n\pi} \sin \frac{n\pi}{2} \sin \frac{n\pi}{3}$
 $C_n = \sqrt{a_n^2 + b_n^2} = \frac{2I_a}{n\pi} \sin \frac{n\pi}{3}$
 $\phi_n = \tan^{-1} (a_n/b_n) = \tan^{-1} (\cot n\pi/2)$
(c) $I_1 = C_1/\sqrt{2} = \sqrt{3I_a}/(\pi\sqrt{2}), \phi_1 = 0$ and $I_s = \sqrt{2} I_a/3$
PF = $I_1/I_s = 3\sqrt{3}/2\pi = 0.827$ and $HF = \sqrt{(I_s/I_1)^2 - 1} = 0.68$

Problem 3-23 (a)

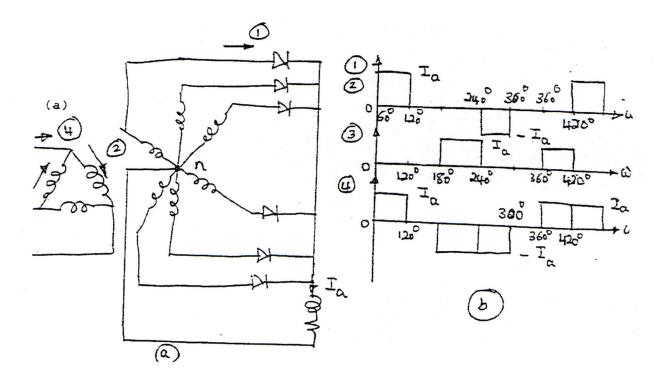


(b) For the primary line current:

$$\begin{aligned} a_{0}/2 &= 0 \\ a_{n} &= \frac{1}{\pi} \int_{k=0}^{k=0}^{k=0} \frac{I_{a}}{\sqrt{3}} \cos(n\theta) d\theta - \frac{1}{\pi} \int_{k=1}^{s=0}^{s=0} \frac{I_{a}}{\sqrt{3}} \cos(n\theta) d\theta \\ &= -\frac{2I_{a}}{\sqrt{3}n\pi} \sin \frac{n\pi}{6} (1 - \cos \frac{2n\pi}{3}) \\ b_{n} &= \frac{1}{\pi} \int_{k=0}^{s=0} \frac{I_{a}}{\sqrt{3}} \sin(n\theta) d\theta - \frac{1}{\pi} \int_{k=1}^{s=0} \frac{I_{a}}{\sqrt{3}} \sin(n\theta) d\theta \\ &= \frac{2I_{a}}{\sqrt{3}n\pi} \cos \frac{n\pi}{6} (1 - \cos \frac{2n\pi}{3}) \\ C_{n} &= \sqrt{a_{n}^{2} + b_{n}^{2}} = \frac{2I_{a}}{\sqrt{3}n\pi} (1 - \cos \frac{2n\pi}{3}) \\ \phi_{n} &= \tan^{-1} (a_{n}/b_{n}) = \tan^{-1} (-\tan n\pi/6) = -n\pi/6 \\ I_{1} &= C_{1}/\sqrt{2} = \sqrt{3I_{a}}/(\pi\sqrt{2}), \ \phi_{1} &= -\pi/6 \ \text{and} \ I_{s} &= \sqrt{2} \ I_{a}/3 \\ \text{(c) For the secondary (or primary) phase current,} \\ a_{0}/2 &= 0 \\ a_{n} &= \frac{1}{\pi} \int_{k=0}^{s=0} \frac{2I_{a}}{3\sqrt{3}} \cos(n\theta) d\theta - \frac{1}{\pi} \int_{s=0}^{s=0} \frac{I_{a}}{3\sqrt{3}} \cos(n\theta) d\theta \\ &= \frac{2I_{a}}{\sqrt{3}n\pi} \cos \frac{n\pi}{2} \sin \frac{n\pi}{3} \\ b_{n} &= \frac{1}{\pi} \int_{k=0}^{s=0} \frac{2I_{a}}{3\sqrt{3}} \sin(n\theta) d\theta - \frac{1}{\pi} \int_{s=0}^{s=0} \frac{I_{a}}{3\sqrt{3}} \sin(n\theta) d\theta \\ &= \frac{2I_{a}}{\sqrt{3}n\pi} \sin \frac{n\pi}{2} \sin \frac{n\pi}{3} \\ C_{n} &= \sqrt{a_{n}^{2} + b_{n}^{2}} = \frac{2I_{a}}{3n\pi} \sin \frac{n\pi}{3} \\ \phi_{n} &= \tan^{-1} (a_{n}/b_{n}) = \tan^{-1} (\cot n\pi/2) = n\pi/2 \\ I_{1} &= C_{1}/\sqrt{2} = I_{a}/(\pi\sqrt{2}), \ \phi_{1} &= 0 \ \text{and} \ I_{s} &= \sqrt{2} \ I_{a}/(3\sqrt{3}) \\ \text{PF} &= I_{1}/I_{s} &= 3\sqrt{3}/2\pi = 0.827 \ \text{and} \ HF &= \sqrt{(I_{s}/I_{1})^{2} - 1} \end{aligned}$$

= 0.68

(a)



(b) For the primary line current:

$$a_{0}/2 = 0$$

$$a_{n} = \frac{I_{a}}{\pi} \Big[\int_{\pi/3}^{2\pi/3} \cos(n\theta) d\theta - \int_{\pi}^{5\pi/3} \cos(n\theta) d\theta + \int_{2\pi}^{2\pi\pi/3} \cos(n\theta) d\theta \Big]$$

$$= -\frac{2I_{a}}{n\pi} \sin \frac{n\pi}{2} \cos \frac{7n\pi}{6}$$

$$b_{n} = \frac{I_{a}}{\pi} \Big[\int_{\pi/3}^{2\pi/3} \sin(n\theta) d\theta - \int_{\pi}^{5\pi/3} \sin(n\theta) d\theta + \int_{2\pi}^{2\pi\pi/3} \sin(n\theta) d\theta \Big]$$

$$= \frac{I_{a}}{n\pi} \Big(1 - \cos n\pi - 2\sin \frac{n\pi}{2} \cos \frac{7n\pi}{6} \Big)$$
For n = 1, C_{1} = $\sqrt{(a_{1}^{2} + b_{1}^{2})} = 2\sqrt{3}I_{a}/\pi$

$$\varphi_{1} = \tan^{-1} (a_{1}/b_{1}) = \tan^{-1}(1/\sqrt{3}) = \pi/6$$

 $I_1 = C_1/2 = \sqrt{2} \sqrt{3}I_a/\pi$, and $I_s = I_a \sqrt{2/3}$ (c) For the secondary or primary phase current, $a_o/2 = 0$

$$a_{n} = \frac{I_{a}}{\pi} \left[\int_{\pi/3}^{2\pi/3} \cos(n\theta) \, d\theta - \int_{4\pi/3}^{5\pi/3} \cos(n\theta) \, d\theta \right] = 0$$

$$b_{n} = \frac{I_{a}}{\pi} \left[\int_{\pi/3}^{2\pi/3} \sin(n\theta) \, d\theta - \int_{4\pi/3}^{5\pi/3} \sin(n\theta) \, d\theta \right]$$

$$= \frac{4I_{a}}{n\pi} \sin \frac{n\pi}{2} \sin \frac{n\pi}{6}$$

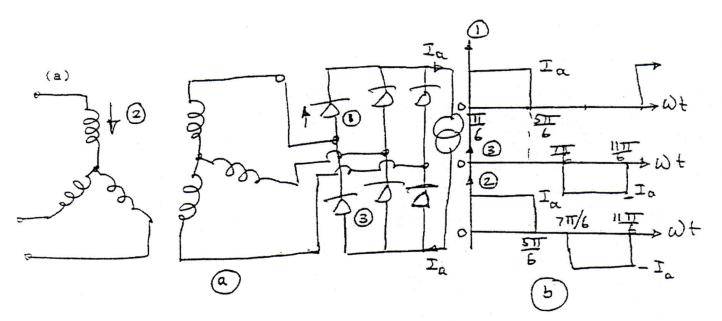
$$C_{n} = b_{n} \text{ and } \phi_{n} = 0$$

$$C_{1} = 2 I_{a}/\pi, I_{1} = C_{1}/\sqrt{2} = 2I_{a}/(\sqrt{2}\pi), \phi_{1} = 0 \text{ and } I_{s} = I_{a}/\sqrt{3}$$

PF = $I_1/I_s = \sqrt{2} \sqrt{3}/\pi = 0.78$ and $HF = \sqrt{(I_s/I_1)^2 - 1} = 0.803$

Problem 3-25

(a)



(b) For the primary (or secondary) phase (or line) current: $a_o/2 = 0$

$$a_{n} = \frac{2I_{a}}{\pi} \int_{\pi/6}^{5\pi/6} \cos(n\theta) d\theta = \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \cos\frac{n\pi}{2}$$

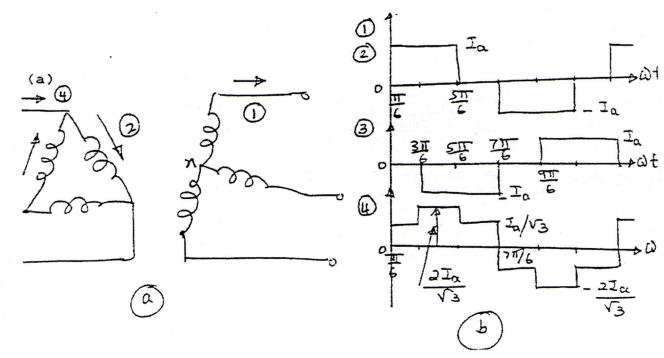
$$b_{n} = \frac{2I_{a}}{\pi} \int_{\pi/6}^{5\pi/6} \sin(n\theta) d\theta = \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \sin\frac{n\pi}{2}$$

$$C_{n} = \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \text{ and } \varphi_{n} = \tan^{-1} (a_{n}/b_{n}) = \tan^{-1}(\cot n\pi/2)$$
(c) $C_{1} = 2\sqrt{3}I_{a}/\pi$ and $\varphi_{1} = 0$
 $I_{1} = C_{1}/\sqrt{2} = \sqrt{2} \sqrt{3}I_{a}/\pi$, and $I_{s} = I_{a} \sqrt{(2/3)}$

$$PF = I_{1}/I_{s} = 3/\pi = 0.9549 \text{ and } HF = \sqrt{(I_{s}/I_{1})^{2} - 1} = 0.3108$$

(a)

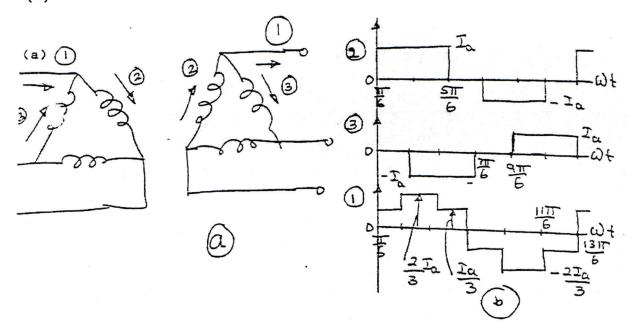
t



(b) For the primary line current, $a_o/2 = 0$

$$\begin{split} a_{n} &= \frac{2I_{a}}{\sqrt{3}\pi} \Big[\int_{\pi}^{\pi/2} \cos(n\theta) \, d\theta + \int_{\pi/2}^{\pi/6} 2\cos(n\theta) \, d\theta + \int_{\pi/6}^{\pi/6} \cos(n\theta) \, d\theta \Big] \\ &= -\frac{8I_{a}}{\sqrt{3}n\pi} \cos\frac{2n\pi}{3} \sin\frac{n\pi}{3} \cos\frac{n\pi}{6} \\ b_{n} &= \frac{2I_{a}}{\sqrt{3}\pi\pi} \Big[\int_{\pi/6}^{\pi/6} \sin(n\theta) \, d\theta + \int_{\pi/2}^{\pi/6} 2\sin(n\theta) \, d\theta + \int_{\pi/6}^{\pi/6} \sin(n\theta) \, d\theta \Big] \\ &= \frac{8I_{a}}{\sqrt{3}n\pi} \sin\frac{2n\pi}{3} \sin\frac{n\pi}{3} \cos\frac{n\pi}{6} \\ C_{n} &= \frac{8I_{a}}{\sqrt{3}n\pi} \sin\frac{n\pi}{3} \cos\frac{n\pi}{6} \\ \phi_{n} &= \tan^{-1} (a_{n}/b_{n}) = \tan^{-1}(\cot(2n\pi/3)) \\ C_{1} &= 2\sqrt{3}I_{a}/\pi \text{ and } I_{1} = C_{1}/\sqrt{2} = \sqrt{2} \sqrt{3}I_{a}/\pi, \, \phi_{1} = \tan^{-1} (-1/\sqrt{3}) = -\pi/6 \\ \text{(c) For the primary (or secondary) phase current,} \\ a_{0}/2 &= 0 \\ a_{n} &= \frac{2I_{a}}{\pi} \int_{\pi/6}^{\pi/6} \cos(n\theta) \, d\theta = \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \cos\frac{n\pi}{2} \\ b_{n} &= \frac{2I_{a}}{\pi} \int_{\pi/6}^{\pi/6} \sin(n\theta) \, d\theta = \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \sin\frac{n\pi}{2} \\ C_{n} &= \frac{4I_{a}}{n\pi} \sin\frac{n\pi}{3} \text{ and } \phi_{n} = \tan^{-1} (a_{n}/b_{n}) = \tan^{-1}(\cot n\pi/2) \\ I_{1} &= C_{1}/\sqrt{2} = \sqrt{2} \sqrt{3}I_{a}/\pi, \, \text{and } I_{s} = I_{a} \sqrt{(2/3)} \\ \text{PF} &= I_{1}/I_{s} &= 3/\pi = 0.9549 \text{ and } HF = \sqrt{(I_{s}/I_{1})^{2} - 1} = 0.3108 \end{split}$$

(a)



(b) For the primary (or secondary) line current, $a_{0}/2 = 0$ $a_{n} = \frac{2I_{a}}{\pi} \int_{\pi/6}^{\pi/6} \cos(n\theta) d\theta = \frac{4I_{a}}{n\pi} \sin \frac{n\pi}{3} \cos \frac{n\pi}{2}$ $b_{n} = \frac{2I_{a}}{\pi} \int_{\pi/6}^{\pi/6} \sin(n\theta) d\theta = \frac{4I_{a}}{n\pi} \sin \frac{n\pi}{3} \sin \frac{n\pi}{2}$ $C_{n} = \frac{4I_{a}}{n\pi} \sin \frac{n\pi}{3} \text{ and } \varphi_{n} = \tan^{-1} (a_{n}/b_{n}) = \tan^{-1}(\cot n\pi/2)$ $I_{1} = C_{1}/\sqrt{2} = \sqrt{2} \sqrt{3}I_{a}/\pi, \text{ and } I_{s} = I_{a} \sqrt{(2/3)}$ (c) For the primary (or secondary) phase current, $a_{0}/2 = 0$ $a_{n} = \frac{2I_{a}}{3\pi} \Big[\int_{\pi/6}^{\pi/2} \cos(n\theta) d\theta + \int_{\pi/2}^{\pi/6} 2\cos(n\theta) d\theta + \int_{\pi/6}^{\pi/6} \cos(n\theta) d\theta \Big]$ $= \frac{8I_{a}}{3n\pi} \cos \frac{2n\pi}{3} \sin \frac{n\pi}{3} \cos \frac{n\pi}{6}$

$$b_{n} = \frac{2I_{a}}{3\pi} \Big[\int_{\pi/6}^{\pi/2} \sin(n\theta) d\theta + \int_{\pi/2}^{6\pi/6} 2\sin(n\theta) d\theta + \int_{5\pi/6}^{7\pi/6} \sin(n\theta) d\theta \Big]$$

$$= \frac{8I_{a}}{3n\pi} \sin \frac{2n\pi}{3} \sin \frac{n\pi}{3} \cos \frac{n\pi}{6}$$

$$C_{n} = \frac{8I_{a}}{3n\pi} \sin \frac{n\pi}{3} \cos \frac{n\pi}{6}$$

$$\phi_{n} = \tan^{-1} (a_{n}/b_{n}) = \tan^{-1} (\cot(2n\pi/3))$$

$$C_{1} = 2\sqrt{3}I_{a}/\pi \text{ and } I_{1} = C_{1}/\sqrt{2} = \sqrt{2} \sqrt{3}I_{a}/\pi, \ \phi_{1} = \tan^{-1} (-1/\sqrt{3}) = -\pi/6$$

$$I_{s} = \sqrt{2}I_{a}/3, \ PF = I_{1}/I_{s} = 3/\pi = 0.9549$$

$$HF = [(I_{s}/I_{1})^{2} - 1]^{\frac{1}{2}} = 0.3108$$

CHAPTER 4

POWER TRANSISTORS

Problem 4-1

 $V_{CC} = 100 \text{ V}, \ \beta_{min} = 10, \ \beta_{max} = 60, \ R_{C} = 5 \ \Omega, \ ODF = 20, \ V_{B} = 8 \text{ V}, \ V_{CE(sat)} = 2.5 \text{ V} \text{ and } V_{BE(sat)} = 1.75 \text{ V}.$ From Eq. (4-14), $I_{CS} = (100 - 2.5)/5 = 19.5 \text{ A}$ From Eq. (4-15), $I_{BS} = 19.5/\beta_{min} = 19.5/10 = 1.95 \text{ A}$ Eq. (4-16) gives the base current for a overdrive factor of 20, $I_{B} = 20 \times 1.95 = 33 \text{ A}$ (a) Eq. (4-9) gives the required value of R_{B} , $R_{B} = (V_{B} - V_{BE(sat)})/I_{B} = (8 - 1.75)/33 = 0.1894 \ \Omega$ (b) From Eq. (4-17), $\beta_{f} = 19.5/33 = 0.59$ (c) Eq. (4-18) yields the total power loss as

 $P_T = 1.75 \times 33 + 2.5 \times 19.5 = 106.5 W$

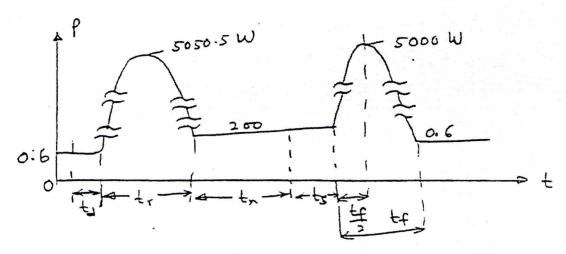
Problem 4-2

$$\begin{split} &V_{CC} = 40 \text{ V}, \ \beta_{min} = 12, \ \beta_{max} = 75, \ R_{C} = 1.5 \ \Omega, \ V_{B} = 6 \text{ V}, \\ &V_{CE(sat)} = 1.2 \text{ V and } V_{BE(sat)} = 1.6 \text{ V}. \\ &From Eq. (4-14), \ I_{CS} = (40 - 1.2)/1.5 = 25.87 \text{ A} \\ &From Eq. (4-15), \ I_{BS} = 25.87/\beta_{min} = 25.87/12 = 2.156 \text{ A} \\ &I_{B} = (6 - 1.6)/0.7 = 4.4/0.7 = 6.286 \text{ A} \\ &(a) \ ODF = I_{B}/I_{BS} = 6.286/2.156 = 2.916 \\ &(b) \ From Eq. (4-17), \ \beta_{f} = 25.8/6.286 = 4.104 \\ &(c) \ Eq. (4-18) \ yields \ the \ total \ power \ loss \ as \\ &P_{T} = 1.2 \times 25.87 + 1.6 \times 6.286 = 41.104 \text{ W} \end{split}$$

Problem 4-3

 $V_{cc} = 200 \text{ V}, \text{ B}_{BE(sat)} = 3 \text{ V}, \text{ I}_{B} = 8 \text{ A}, \text{ V}_{CE(sat)} = 2 \text{ V}, \text{ I}_{CS} = 100 \text{ A}, \text{ t}_{d} = 0.5 \text{ }\mu\text{s}, \text{ t}_{r}$ = 1 μ s, t_s = 5 μ s, t_f = 3 μ s, f_s = 10 kHz, k = 0.5, T = 1/f_s = 100 μ s. kT = t_d + $t_r + t_n = 50 \ \mu s$ and $t_n = 50 - 0.5 - 1 = 48.5 \ \mu s$, $(1-k)T = t_s + t_f + t_o = 50 \ \mu s$ and $t_0 = 50 - 5 - 3 = 42 \ \mu s$. (a) From Eq. (4-21), $P_{c}(t) = 3 \times 10^{-3} \times 200 = 0.6 W$ $P_d = 3 \times 10^{-3} \times 200 \times 0.5 \times 10^{-6} \times 10 \times 10^3 = 3 \text{ mW}$ During rise time, $0 \le t \le t_r$: From Eq. (4-23), $t_m = 1 \times 200/[2(200 - 2)] = 0.505 \,\mu s$ From Eq. (4-24), $P_p = 200^2 \times 100/[4(200 - 2)] = 5050.5 \text{ W}$ From Eq. (4-25), $P_r = 10 \times 10^3 \times 100 \times 1 \times 10^{-6} [220/2 + (2 - 200)/3] = 34 W$ $P_{on} = P_d + P_r = 0.003 + 34 = 34.003 W$ (b) Conduction Period, $0 \le t \le t_n$: $P_{c}(t) = 2 \times 100 = 200 W$ From Eq. (4-27), $P_n = 2 \times 100 \times 48.5 \times 10^{-6} \times 10 \times 10^3 = 97 \text{ W}$ (c) Storage Period, $0 \le t \le t_s$: $P_{c}(t) = 2 \times 100 = 200 W$ From Eq. (4-28), $P_s = 2 \times 100 \times 5 \times 10^{-6} \times 10 \times 10^{3} = 10 \text{ W}$ Fall time, $0 \le t \le t_f$: From Eq. (4-30), $P_p = 200 \times 100/4 = 5000 W$ From Eq. (4-31), $P_f = 200 \times 100 \times 3 \times 10^{-6} \times 10 \times 10^{3}/6 = 100 \text{ W}$ $P_{off} = P_s + P_f = 10 + 100 = 110 W$ (d) Off-period, $0 \le t \le t_o$: $P_c(t) = 3 \times 10^{-3} \times 200 = 0.6 W$

From Eq. (4-33), $P_o = 3 \times 10^{-3} \times 200 \times 42 \times 10^{-6} \times 10 \times 10^3 = 0.252 \text{ W}$ (e) The total power loss in the transistor due to collector current is $P_T = P_{on} + P_n + P_{off} + P_o = 34.003 + 97 + 110 + 241.255 \text{ W}$ (f)



Problem 4-4

 $T_{J} = 150 \text{ °C}, T_{A} = 25 \text{ °C}, R_{JC} = 0.04 \text{ °C}/W, R_{CS} = 0.05 \text{ °C}/W$ From Problem 4-3, $P_{T} = 241.25 \text{ W}$ $P_{T}(R_{JC} + R_{CS} + R_{SA}) = T_{J} - T_{A} = 150 - 25 = 125$ $R_{SA} = 125/241.25 - 0.04 - 0.05 = 0.0681 \text{ °C}/W$

Problem 4-5

$$\begin{split} \mathsf{B}_{\mathsf{BE}(\mathsf{sat})} &= 3 \text{ V}, \ I_{\mathsf{B}} = 8 \text{ A}, \ \mathsf{T} = 1/f_{\mathsf{s}} = 100 \ \mu\mathsf{s}, \ \mathsf{k} = 0.5, \ \mathsf{kT} = 50 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{d}} = 0.5 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{r}} \\ &= 1 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{n}} = 50 \ - 1.5 \ = 48.5 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{s}} = 5 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{f}} = 3 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{on}} = \mathsf{t}_{\mathsf{d}} + \mathsf{t}_{\mathsf{r}} = 1.5 \ \mu\mathsf{s}, \ \mathsf{t}_{\mathsf{off}} \\ &= \mathsf{t}_{\mathsf{s}} + \mathsf{t}_{\mathsf{f}} = 5 + 3 = 8 \ \mu\mathsf{s} \\ (a) \ \mathsf{During the period}, \ 0 \le \mathsf{t} \le (\mathsf{t}_{\mathsf{on}} + \mathsf{t}_{\mathsf{n}}): \\ &i_{\mathsf{b}}(\mathsf{t}) = I_{\mathsf{BS}} \end{split}$$

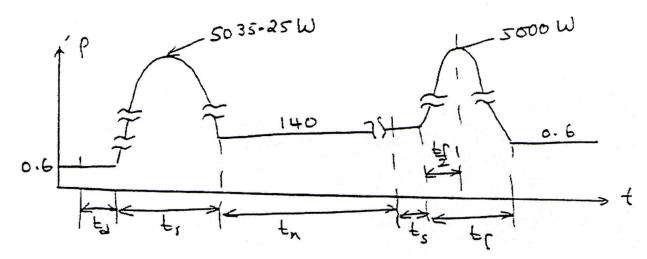
 $v_{BE}(t) = V_{BE(sat)}$

The instantaneous power due to the base current is $P_{b}(t) = i_{b} v_{BE} = I_{BS} V_{BE(sat)} = 8 \times 3 = 24 W$ During the period, $0 \le t \le t_{o} = (T - t_{on} - t_{n})$: $P_{b}(t) = 0$ From Eq. (4-35), the average power loss is $P_{B} = I_{BS} V_{BE(sat)} (t_{on} + t_{n} + t_{s})f_{s}$ $= 8 \times 3 \times (1.5 + 48.5 + 5) \times 10^{-6} \times 10 \times 10^{3} = 13.2 W$

Problem 4-6

 $V_{cc} = 200 V$, $B_{BE(sat)} = 2.3 V$, $I_B = 8 A$, $V_{CE(sat)} = 1.4 V$, $I_{CS} = 100 A$, $t_d = 0.1$ μ s, t_r = 0.45 μ s, t_s = 3.2 μ s, t_f = 1.1 μ s, f_s = 10 kHz, k = 0.5, T = 1/f_s = 100 μ s. kT = t_d + t_r + t_n = 50 μ s and t_n = 50 - 0.45 - 0.1 = 49.45 μ s, (1-k)T = t_s + t_f + t_o = 50 µs and t_o = 50 - 3.2 - 1.1 = 45.7 µs. (a) From Eq. (4-21), $P_c(t) = 3 \times 10^{-3} \times 200 = 0.6 W$ $P_d = 3 \times 10^{-3} \times 200 \times 0.1 \times 10^{-6} \times 10 \times 10^3 = 0.6 \text{ mW}$ During rise time, $0 \le t \le t_r$: From Eq. (4-23), $t_m = 0.45 \times 200/[2(200 - 1.4)] = 0.2266 \Box s$ From Eq. (4-24), $P_p = 200^2 \times 100/[4(200 - 1.4)] = 5035.25 \text{ W}$ From Eq. (4-25), $P_r = 10 \times 10^3 \times 100 \times 0.45 \times 10^{-6} [220/2 + (1.4 - 200)/3] = 15.21 \text{ W}$ $P_{on} = P_d + P_r = 0.0006 + 15.21 W = 15.21 W$ (b) Conduction Period, $0 \le t \le t_n$: $P_{c}(t) = 1.4 \times 100 = 280 W$ From Eq. (4-27), $P_n = 1.4 \times 100 \times 49.45 \times 10^{-6} \times 10 \times 10^3 = 69.23 \text{ W}$ (c) Storage Period, $0 \le t \le t_s$: $P_{c}(t) = 1.4 \times 100 = 140 W$

From Eq. (4-28), $P_s = 1.4 \times 100 \times 3.2 \times 10^{-6} \times 10 \times 10^3 = 4.48 \text{ W}$ Fall time, $0 \le t \le t_f$: From Eq. (4-30), $P_p = 200 \times 100/4 = 5000 \text{ W}$ From Eq. (4-31), $P_f = 200 \times 100 \times 1.1 \times 10^{-6} \times 10 \times 10^3/6 = 36.67 \text{ W}$ $P_{off} = P_s + P_f = 4.48 + 36.67 = 41.15 \text{ W}$ (d) Off-period, $0 \le t \le t_o$: $P_c(t) = 3 \times 10^{-3} \times 200 = 0.6 \text{ W}$ From Eq. (4-33), $P_o = 3 \times 10^{-3} \times 200 \times 45.7 \times 10^{-6} \times 10 \times 10^3 = 0.274 \text{ W}$ (e) The total power loss in the transistor due to collector current is $P_T = P_{on} + P_n + P_{off} + P_o = 15.21 + 69.23 + 41.15 + 0.274 = 125.87 \text{ W}$ (f)



Problem 4-7

 $\begin{array}{l} V_{DD}=40 \ \text{V}, \ I_{D}=35 \ \text{A}, \ I_{DSS}=250 \ \mu\text{A}, \ \text{R}_{DS}=28 \ \text{m}\Omega, \ \text{V}_{GS}=10 \ \text{V}, \ t_{d(on)}=25 \\ \text{ns, } t_{r}=60 \ \text{ns, } t_{d(off)}=70 \ \text{ns, } t_{f}=25 \ \text{ns, } f_{s}=20 \ \text{kHz}, \ \text{k}=0.6, \ \text{T}=1/f_{s}=50 \\ \mu\text{s. } \text{kT}=t_{d(on)}+t_{r}+t_{n}=50000 \ \text{ns and } t_{n}=50000 \ \text{-}\ 25 \ \text{-}\ 60=29915 \ \text{ns, } (1-k) \\ \text{K}=t_{f}+t_{d(off)}+t_{o}=20000 \ \text{ns and } t_{o}=20000 \ \text{-}\ 70 \ \text{-}\ 25=19905 \ \text{ns.} \\ \end{tabular}$

 $i_D(t) = I_{DSS}$ $v_{DS}(t) = V_{DD}$

The instantaneous power due to the collector current is $P_D(t) = i_D v_{DS} = I_{DSS} V_{DD} = 250 \times 10^{-3} \times 40 = 0.01 W$ The average power loss during the delay time is From Eq. (4-21), $P_d = I_{DSS} V_{DD} t_{d(on)} f_s$

= 250 x 10^{-3} x 40 x 25 x 10^{-9} x 20 x 10^{3} = 5 mW During rise time, 0 $\leq t \leq t_r$:

$$i_{D}(t) = \frac{I_{DS}}{t_{r}}t$$

$$v_{DS}(t) = V_{DD} + (R_{DS}I_{DS} - V_{DD})\frac{t}{t_{r}}$$

From Eq. (4-22)

$$P_{D}(t) = i_{D} v_{DS} = \frac{I_{DS}}{t_{r}} t \left[V_{DD} + (R_{DS} I_{DS} - V_{DD}) \frac{t}{t_{r}} \right]$$

From Eq. (4-23) the power $P_D(t)$ will be maximum when $t = t_m$, where

$$t_m = \frac{t_r V_{DD}}{2(V_{DD} - R_{DS}I_{DS})} = \frac{60 \times 10^{-9} \times 40}{2 \times (40 - 25 \times 10^{-3} \times 35)} = 30.67 \text{ ns}$$

and Eq. (4-24) yields the peak power

$$P_p = \frac{V_{DD}^2 I_{DS}}{4(V_{CC} - V_{CE(sat)})} = \frac{40^2 \text{ x } 35}{4(40 - 25 \text{ x } 10^{-3} \text{ x } 35)} = 357.83 \text{ W}$$

From Eq. (4-25).

$$P_r = f_S I_{DS} t_r \left[\frac{V_{DD}}{2} + \frac{R_{DS} I_{DS} - V_{CC}}{3} \right]$$

= $20 \times 10^3 \times 35 \times 60 \times 10^{-9} [40/2 + (25 \times 10^{-3} \times 35 - 40)/3] = 1.3877$ W The total power loss during the turn-on is $P_{on} = P_d + P_r = 0.005 + 1.3877 = 1.39275$ W (b) Conduction Period, $0 \le t \le t_n$: $i_D(t) = I_{DS}$ $v_{DS}(t) = R_{DS} I_{DS}$ $P_D(t) = i_D v_{DS} = R_{DS} I_{DS} I_{DS} = 25 \times 10^{-3} \times 35^2 = 30.625 W$ From Eq. (4-27), $P_n = R_{DS} I_{DS} I_{DS} t_n f_s$ $= 25 \times 10^{-3} \times 35^2 \times 29915 \times 10^{-9} \times 20 \times 10^3 = 18.32 W$ (c) Storage Period, $0 \le t \le t_{d(off)}$: $i_D(t) = I_{DS}$ $v_{DS}(t) = R_{DS} I_{DS}$ $P_c(t) = i_D v_{DS} = R_{DS} I_{DS} I_{DS} = 25 \times 10^{-3} \times 35^2 = 30.625 W$ $P_{D(off)} = R_{DS} I_{DS} I_{DS} t_{d(off)} f_s$

= 25 x 10⁻³ x35² x 70 x 10⁻⁹ x 20 x 10³ = 42.87 mW Fall time, 0 \leq t \leq t_f:

$$i_{D}(t) = I_{DS} \left(1 - \frac{t}{t_{f}} \right)$$
$$v_{DS}(t) = \frac{V_{DD}}{t_{f}} t$$
$$P_{D}(t) = V_{DD} I_{DS} \left[\left(1 - \frac{t}{t_{f}} \right) \frac{t}{t_{f}} \right]$$

This power loss during fall time will be maximum when $t = t_f/2 = 12.5$ ns. From Eq. (4-30), the peak power,

 $P_D = V_{DD} I_{DS} / 4 = 40 \times 35 / 4 = 350 W$ From Eq. (4-31), $P_f = V_{DD} I_{DS} t_f f_s / 6$

 $= 40 \times 35 \times 25 \times 10^{-9} \times 20 \times 10^{3}/6 = 0.117 \text{ W}$

The power loss during turn-off is

 $P_{off} = P_{D(off)} + P_f = \ 0.04287 + 0.117 = 0.15987 \ W$ (d) Off-period, $0 \le t \le t_o$:

$$\begin{split} i_D(t) &= I_{DSS} \\ v_{DS}(t) &= V_{DD} \\ P_D(t) &= i_D v_{DS} = I_{DSS} V_{DD} = 250 \times 10^{-6} \times 40 = 10 \text{ mW} \\ P_o &= I_{DSS} V_{DD} t_o f_s \\ &= 250 \times 10^{-6} \times 40 \times 19905 \times 10^{-9} \times 20 \times 10^3 = 3.981 \text{ mW} \\ (e) \text{ The total power loss in the transistor due to collector current is} \\ P_T &= P_{on} + P_n + P_{off} + P_o \\ &= 1.3927 + 18.32 + 0.04287 + 0.01 = 20.466 \text{ W} \end{split}$$

Problem 4-8

From Problem 4-7, $P_T = 20.466 \text{ W}$ $R_{JC} = 1 \text{ }^{\circ}\text{K/W}, R_{CS} = 1 \text{ }^{\circ}\text{K/W}, T_J = 150 \text{ }^{\circ}\text{C}, T_A = 30 \text{ }^{\circ}\text{C}$ $T_J = 150 \text{ }^{\circ}\text{C} + 273 = 423 \text{ }^{\circ}\text{K}$ $T_A = 30 \text{ }^{\circ}\text{C} + 273 = 303 \text{ }^{\circ}\text{K}$ $P_T(R_{JC} + R_{CS} + R_{SA}) = T_J - T_A = 423 - 303 = 120$ $R_{SA} = 120/20.466 - 1 - 1 = 3.863 \text{ }^{\circ}\text{K/W}$

Problem 4-9

$$\begin{split} I_T &= 200 \text{ A}, \text{ V}_{CE1} = 1.5 \text{ V}, \text{ V}_{CE2} = 1.1 \text{ V} \\ (a) \text{ R}_{e1} &= 10 \text{ m}\Omega, \text{ R}_{e2} = 20 \text{ m}\Omega \\ I_{E1} + I_{E2} &= I_T \\ \text{ V}_{CE1} + I_{E1} \text{ R}_{e1} &= \text{ V}_{CE2} + I_{E2} \text{ R}_{e2} \\ \text{or } I_{E1} &= (\text{V}_{CE2} - \text{V}_{CE1} + I_T \text{ R}_{e2})/(\text{R}_{e1} + \text{R}_{e2}) \\ &= (1.1 - 1.5 + 200 \times 20 \times 10^{-3})/(0.01 + 0.02) = 120 \text{ A or } 60 \% \\ I_{E2} &= 200 - 120 = 80 \text{ A or } 40 \%. \quad \Delta I = 60 - 40 = 20 \% \\ (b) \text{ R}_{e1} &= \text{R}_{e2} = 20 \text{ m}\Omega \end{split}$$

 $I_{E1} = (1.1 - 1.5 + 200 \times 20 \times 10^{-3})/(0.01 + 0.02) = 90 \text{ A or } 45 \%$ $I_{E2} = 200 - 90 = 110 \text{ A or } 55 \%$. $\Delta I = 55 - 45 = 10 \%$

Problem 4-10

$$\begin{split} &I_L = 100 \text{ A}, \text{ V}_s = 400 \text{ V}, \text{ f}_s = 20 \text{ kHz}, \text{ t}_r = 1 \text{ }\mu\text{s}, \text{ and } \text{ t}_f = 3 \text{ }\mu\text{s}. \end{split}$$

Problem 4-11

$$\begin{split} I_L &= 40 \text{ A}_r \text{ V}_s = 30 \text{ V}, \text{ f}_s = 50 \text{ kHz}, \text{ t}_r = 60 \text{ ns}, \text{ and } \text{ t}_f = 25 \text{ ns} \\ \text{(a) From Eq. (4-44), } L_s &= 30 \times 60 \times 10^{-9}/40 = 0.045 \ \mu\text{H} \\ \text{(b) From Eq. (4-46), } C_s &= 40 \times 25 \times 10^{-9}/30 = 0.0333 \ \mu\text{F} \\ \text{(c) From Eq. (4-47), } R_s &= 2 \ \sqrt{(45/33.33)} = 2.324 \ \Omega \\ \text{(d) From Eq. (4-48), } R_s &= 10^6/(3 \times 50 \times 33.33 = 200 \ \Omega \\ \text{(e) } \text{ V}_s/\text{R}_s &= 0.05 \times \text{I}_L \text{ or } 30/\text{R}_s = 0.05 \times 40 \text{ or } \text{R}_s = 15 \ \Omega \\ \text{(f) From Eq. (4-49), the snubber loss is} \\ P_s &= 0.5 \text{ C}_s \text{ V}_s^2 \text{ f}_s \\ &= 0.5 \times 33.33 \times 10^{-9} \times 30^2 \times 50 \times 10^3 = 0.75 \text{ W} \end{split}$$

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CHAPTER 5

DC-DC CONVERTERS

Problem 5-1

$$\begin{split} V_s &= 220 \text{ V}, \text{ k} = 0.8, \text{ R} = 20 \ \Omega \text{ and } \text{v}_{ch} = 1.5 \text{ V}. \\ \text{(a) From Eq. (5-1), } V_a &= 0.8 \text{ x} (220 - 1.5) = 174.8 \text{ V} \\ \text{(b) From Eq. (5-2), } V_o &= \sqrt{0.8 \text{ x} 220} = \text{V} \\ \text{(c) From Eq. (5-5), } P_o &= 0.8 \text{ x} (220 - 1.5)^2 / 20 = 3819.4 \text{ W} \\ \text{From Eq. (5-6), } P_i &= 0.8 \text{ x} 220 \text{ x} (220 - 1.5) / 20 = 3845.6 \text{ W} \\ \text{The chopper efficiency is } P_o / P_i &= 3819.4 / 3845.6 = 99.32 \% \\ \text{(d) From Eq. (5-4), } R_i &= 20 / 0.8 = 12.5 \ \Omega \\ \text{(e) From Eq. (5-8), the output voltage is} \end{split}$$

$$v_o(t) = \frac{220}{\pi} \left[\sin(2\pi \times 0.8) \cos(2\pi \times 10000t) + 0.691 \times \sin(2\pi \times 10000t) \right]$$

= 82.32 \times \sin(62832t + \phi)

where $\phi = \tan^{-1} [\sin(0.8 \times 2\pi)/0.691] = 54^{\circ}$.

The rms value is $V_1 = 82.32/\sqrt{2} = 58.2 \text{ V}$

<u>Note</u>: The efficiency calculation, which includes the conduction loss of the chopper, does not take into account the switching loss due to turn-on and turn-off of the converter.

Problem 5-2

 $V_{s} = 220 \text{ V}, \text{ R} = 10 \ \Omega, \text{ L} = 15.5 \text{ mH}, \text{ E} = 20 \text{ V}, \text{ k} = 0.5 \text{ and } \text{f} = 5000 \text{ Hz}$ From Eq. (5-15), I₂ = 0.9375 I₁ + 1.2496 From Eq. (5-16), I₁ = 0.9375 I₂ - 1.2496 (a) Solving these two equations, I₁ = 8.6453 A (b) $I_2 = 9.3547 A$

(c) $\Delta I = I_2 - I_1 = 9.35475 - 8.6453 = 0.7094 \text{ A}$ From Eq. (5-21), $\Delta I_{max} = 0.7094 \text{ A}$ and Eq. (5-22) gives the approximately value, $\Delta I_{max} = 0.7097 \text{ A}$ (d) The average load current is approximately,

 $I_a = (I_2 + I_1)/2 = (9.35475 + 8.6453)/2 = 9 A$ (e) From Eq. (5-24),

$$I_o = \left[I_1^2 + \frac{\left(I_2 - I_1\right)^2}{3} + I_1 \left(I_2 - I_1\right) \right]^{1/2} = 9.002 \text{ A}$$

(f) $I_s = k I_a = 0.8 \times 9 = 7.2 \text{ A}$ and the input resistance is $R_i = V_s/I_s = 220/7.2 = 30.56 \Omega$ (g) From Eq. (5-25), $I_R = \sqrt{k} I_o = \sqrt{0.8 \times 22.1} = 15.63 \text{ A}$

Problem 5-3

 $V_s = 220 V, R = 0.2 \Omega, E = 10 V, f = 200 Hz, T = 1/f = 0.005 s$ $\Delta i = 200 \times 0.5 = 10 A.$

$$V_a = k V_s = R I_a$$

The voltage across the inductor is given by

$$L\frac{di}{dt} = V_S - RI_a = V_S - kV_S = (1-k)V_S$$

For a linear rise of current, $dt = t_1 = kT$ and $di = \Delta i$

$$\Delta i = \frac{(1-k) V_S}{L} k T$$

For worst case ripple condition: $\frac{d(\Delta i)}{dk} = 0$

and this gives, k = 0.5

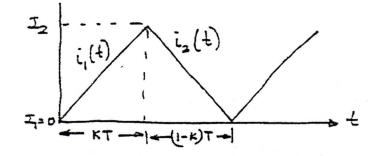
 $\Delta i L = 10 \times L = 220 (1 - 0.5) 0.5 \times 0.005$ or L = 27.5 mH

 $V_{s} = 110 \text{ V}, \text{ E} = 220 \text{ V}, P_{o} = 30 \text{ kW} = 30000 \text{ W}$ (c) Since the input power must be the same as the output power, $V_{s} I_{s} = P_{o} \text{ or } 110 \text{ x } I_{s} = 30000 \text{ or } I_{s} = A$ (a) The battery current, $I_{b} = P_{o}/\text{E} = 30000/220 = 136.36 \text{ A}$ $I_{b} = (1 - \text{k}) I_{s} \text{ or } \text{k} = 136.36/272.73 - 1 = 0.5$ (b) $R_{ch} = (1 - \text{k}) \text{ E}/I_{s} = (1 - 0.5) \text{ x } 220/272.73 = 0.4033 \Omega$

Problem 5-5

 $V_{s} = 110 \text{ V}, \text{ L} = 7.5 \text{ mH}, \text{ E} = 220 \text{ V}$ From Eq. (5-28), $i_{1}(t) = (110 \times 10^{3}/7.5) t + I_{1}$ From Eq. (5-29), $i_{2}(t) = [(110 - 220) \times 10^{3}/7.5) t + I_{2} = -(110 \times 10^{3}/7.5) t + I_{2}$ where $I_{2} = i_{1}(t=kT) = (110 \times 10^{3}/7.5) kT + I_{1}$ $I_{1} = i_{2}[t=(1-k) kT] = -110 \times 10^{3}/7.5) (1-k)kT + I_{2}$

Solving for I₁ and I₂ yields I₁ = 0, I₂ = (110 x $10^3/7.5$) kT $i_1(t) = (110 x <math>10^3/7.5)t$, for $0 \le t \le kT$ $i_2(t) = -(110 x <math>10^3/7.5)t + (110 x <math>10^3/7.5)(1-k)T$, for $0 \le t \le (1 - k)T$.



 V_s = 600 V, R = 0.25 $\Omega,$ L = 20 mH, E = 150 V, k = 0.1 to 0.9 and f = 250 Hz For k=0.1, the load current is discontinuous From Eq. (5-15), $I_2 = 8.977$ From Eq. (5-16), $I_1 = 0$, $\Delta I = 8.977$ A and $I_a = 4.4885$ A For k=0.2, the load current is discontinuous I_2 = 17.9103 A, I_1 = 0 A, ΔI = 17.9103 A and I_a = 8.955 A For k = 0.3 $I_2 = 0.9851 I_1 + 26.7985, I_1 = 0.9656 I_2 - 20.6367$ $I_2 = 132.64 \text{ A}, I_1 = 107.44 \text{ A}, \Delta I = 25.2 \text{ A} \text{ and } I_a = 120.04 \text{ A}$ For k = 0.4 $I_2 = 0.9802 I_1 + 35.64, I_1 = 0.97044 I_2 - 17.733$ $I_2 = 374.42 \text{ A}, I_1 = 345.62 \text{ A}, \Delta I = 28.8 \text{ A} \text{ and } I_a = 360.02 \text{ A}$ For k = 0.5 $I_2 = 0.9753 I_1 + 44.44, I_1 = 0.97045 I_2 - 14.814$ $I_2 = 615 \text{ A}, I_1 = 585 \text{ A}, \Delta I = 30 \text{ A} \text{ and } I_a = 600 \text{ A}$ For k = 0.6 $I_2 = 0.97044 I_1 + 53.2, I_1 = 0.9802 I_2 - 11.881$ $I_2 = 854.38 \text{ A}, I_1 = 825.58 \text{ A}, \Delta I = 28.8 \text{ A} \text{ and } I_a = 840 \text{ A}$ For k = 0.7 $I_2 = 0.9656 I_1 + 61.91, I_1 = 0.9851 I_2 - 8.933$ $I_2 = 1092.6 \text{ A}, I_1 = 1067.4 \text{ A}, \Delta I = 25.2 \text{ A} \text{ and } I_a = 1080 \text{ A}$ For k = 0.8 $I_2 = 0.9608 I_1 + 70.58, I_1 = 0.99 I_2 - 5.97$ I_2 = 1329.6 A, I_1 = 1310.4 A, ΔI = 19.2 A and I_a = 1320 A For k = 0.9

$$\begin{split} I_2 &= 0.956 \ I_1 + \ 79.2, \ I_1 = 0.995 \ I_2 - 2.99 \\ I_2 &= 1565.4 \ \text{A}, \ I_1 = 1554.6 \ \text{A}, \ \Delta I = 10.8 \ \text{A} \ \text{and} \ I_a = 1560 \ \text{A} \end{split}$$

Problem 5-7

 V_s = 600 V, R = 0.25 $\Omega,$ L = 20 mH, E = 150 V, k = 0.1 to 0.9 and f = 250 Hz

The maximum ripple occurs at k = 0.5. From Eq. (5-21), $\Delta I_{max} = (600/0.25) \tanh [0.25/(4 \times 250 \times 0.02)] = 29.9985 A.$

From Eq. (5-22), $\Delta I_{max} = [600/(4 \times 250 \times 0.02)] = 30 \text{ A}$

Problem 5-8

$$\begin{split} &V_{s} = 10 \text{ V, } f = 1 \text{ kHz, } R = 10 \text{ }\Omega\text{, } L = 6.5 \text{ mH, } E = 5 \text{ V and } k = 0.5. \\ &V_{s} \coloneqq 10 \qquad R \coloneqq 10 \qquad L \coloneqq 6.5 \cdot 10^{-3} \qquad f \coloneqq 1000 \\ &E \coloneqq 5 \qquad k \coloneqq 0.5 \qquad T \coloneqq \frac{1}{f} \qquad z \coloneqq \frac{T \cdot R}{L} \qquad z = 1.54 \\ &From \text{ Eq. (5-35), we get} \\ &I_{1} \coloneqq \frac{V_{s} \cdot k \cdot z}{R} \cdot \frac{e^{-(1-k) \cdot z}}{1-e^{-(1-k) \cdot z}} + \frac{V_{s} - E}{R} \qquad I_{1} = 1.16 \quad \text{ A} \\ &From \text{ Eq. (5-36), we get} \\ &I_{2} \coloneqq \frac{V_{s} \cdot k \cdot z}{R} \cdot \frac{1}{1-e^{-(1-k) \cdot z}} + \frac{V_{s} - E}{R} \qquad I_{2} = 1.93 \quad \text{ A} \\ &\Delta I \coloneqq I_{2} - I_{1} \qquad \Delta I = 0.77 \quad \text{ A} \end{split}$$

 $V_{s} = 15 \text{ V}, \Delta V_{c} = 10 \text{ mV}, \Delta I = 0.5 \text{ A}, \text{ f} = 20 \text{ kHz}, V_{a} = 5 \text{ V} \text{ and } I_{a} = 0.5 \text{ A}$ (a) From Eq. (5-48), $V_{a} = k \text{ V}_{s}$ and $k = V_{a}/V_{s} = 5/15 = 0.3333$ (b) From Eq. (5-52), $L = 5 (15 - 5)/(0.5 \times 20000 \times 15) = 333.3 \mu\text{H}$ (c) From Eq. (5-53), $C = 0.5/(8 \times 10 \times 10^{-3} \times 20000) = 312.5 \mu\text{F}$

(d)
$$R := \frac{V_a}{I_a}$$
 $R = 10$

From Eq. (5-56) $L_{c}(k) := \frac{(1-k)\cdot R}{2\cdot f}$ $L_{c}(0.333)\cdot 10^{6} = 166.75 \ \mu H$ From Eq. (5-89) $C_{c}(k) := \frac{1-k}{16\cdot L_{c}(0.333)\cdot f^{2}}$ $C_{c}(0.333)\cdot 10^{6} = 0.63 \ \mu F$

Problem 5-10

(e)
$$R := \frac{V_a}{I_a}$$
 $R = 30$

From Eq. (5-72)	$L_{c}(k) := \frac{k \cdot (1-k) \cdot R}{2 \cdot f}$	$L_c(0.6) \cdot 10^6 = 180$	μH
From Eq. (5-73)	$C_{c}(k) := \frac{k}{2 \cdot f \cdot R}$	$C_c(0.6) \cdot 10^6 = 0.5$	μF

- $V_s = 12 \text{ V}, \text{ } \text{k} = 0.6, \text{ I}_a = 1.5 \text{ A}, \text{ } \text{f} = 25 \text{ } \text{kHz}, \text{ } \text{L} = 250 \text{ } \mu\text{H}, \text{ } \text{and } \text{C} = 220 \text{ } \mu\text{F}$ (a) From Eq. (5-78), $V_a = -12 \text{ } \text{x} \text{ } 0.6/(1 \text{ } 0.6) = -18 \text{ } \text{V}$
- (b) From Eq. (5-87), the peak-to-peak output ripple voltage is
- $\Delta V_c = 1.5 \times 0.6 / (25000 \times 220 \times 10^{-6}) = 163.64 \text{ mV}$

(c) From Eq. (5-84), the peak-to-peak inductor ripple voltage is

 $\Delta I = 12 \times 0.6/(25000 \times 250 \times 10^{-6}) = 1.152 \text{ A}$

(d) From Eq. (5-81), $I_s = 1.5 \times 0.6/(1 - 0.6) = 2.25 \text{ A}$

Since I_s is the average of duration kT, the peak to peak current of transistor, $I_p = I_s/k + \Delta I/2 = 2.25/0.6 + 1.152/2 = 4.326 \text{ A}$

(e)
$$R := \frac{-V_a}{I_a}$$
 $R = \bullet$
From Eq. (5-88) $L_c(k) := \frac{(1-k) \cdot R}{2 \cdot f}$ $L_c(0.6) \cdot 10^6 = \bullet$ μH

From Eq. (5-89) $C_c(k) := \frac{k}{2 \cdot f \cdot R}$ $C_c(0.6) \cdot 10^6 = \bullet$ μF

Problem 5-12

 V_s = 15 V, k = 0.4, I_a = 1.25 A, f = 25 kHz, L_1 = 250 $\mu H,$ C_1 = 400 $\mu F,$ L_2 = 350 μH and C_2 = 220 μF

(a) From Eq. (5-100), $V_a = -0.4 \times 15/(1 - 0.4) = -10 V$ (b) From Eq. (5-103), $I_s = 1.25 \times 0.4/(1 - 0.4) = 0.83 A$ (c) From Eq. (5-106), $\Delta I_1 = 15 \times 0.4/(25000 \times 250 \times 10^{-6}) = 0.96 A$ (d) From Eq. (5-112), $\Delta V_{c1} = 0.83 (1 - 0.4)/(25000 \times 400 \times 10^{-6}) = 50 mV$ (e) From Eq. (5-109), $\Delta I_2 = 0.4 \times 15/(25000 \times 350 \times 10^{-6}) = 0.69 A$ (f) From Eq. (5-113), $\Delta V_{c2} = 0.69/(8 \times 25000 \times 220 \times 10^{-6}) = 15.58 mV$ (g) From Eq. (5-120), $\Delta I_{L2} = 1.25/(1.0 - 2 \times 0.4) = 6.25 A$ $I_p = I_s + I_1/2 + I_{L2} + \Delta I_2/2 = 0.83 + 0.96/2 + 6.25 + 0.69/2 = 7.91 A$

Problem 5-13

 V_s = 15 V, k = 0.4, I_a = 1.25 A, f = 25 kHz, L_1 = 250 $\mu H,$ C_1 = 400 $\mu F,$ L_2 = 350 μH and C_2 = 220 μF

 $V_s := 15$ k := 0.4 $I_a := 1.25$ $f := 25 \cdot 10^3$

From Eq. (5-115) $L_{c1}(k) := \frac{(1-k) \cdot R^2}{2 \cdot k \cdot f}$ $L_{c1}(0.4) \cdot 1000 = 4.32 \text{ mH}$

- From Eq. (5-116) $L_{c2}(k) := \frac{(1-k) \cdot R}{2 \cdot f}$ $L_{c2}(0.4) \cdot 1000 = 0.14$ mH
- From Eq. (5-117) $C_{c1}(k) := \frac{k}{2 \cdot f \cdot R}$ $C_{c1}(0.5) \cdot 10^6 = 0.83$ μF
- From Eq. (5-118) $C_{c2}(k) := \frac{1}{8 \cdot f \cdot R}$ $C_{c2}(0.5) \cdot 10^6 = 0.42$ μF

Problem 5-14

 $V_s = 110 V$, $V_a = 80 V$, $I_a = 20 A$

$$\begin{split} &\Delta V_c = 0.05 \times V_a = 0.05 \times 80 = 4 \ V \\ &R = V_a/I_a = 80/20 = 4 \ \Omega \\ &From Eq. (5-48), \ &k = V_a/V_s = 80/110 = 0.7273 \\ &From Eq. (5-49) \ I_s = k \ I_a = 0.7273 \times 20 = 14.55 \ A \\ &\Delta I_L = 0.025 \times I_a = 0.025 \times 20 = 0.5 \ A \\ &\Delta I = 0.1 \times I_a = 0.1 \times 20 = 2 \ A \\ &(a) \ From Eq. (5-51), \ we \ get \ the \ value \ of \ L_e \end{split}$$

$$L_{e} = \frac{V_{a} (V_{S} - V_{a})}{\Delta I \ f \ V_{S}} = \frac{80 \times (110 - 80)}{2 \times 10 \ kHz \ \times 110} = 1.091 \ \text{mH}$$

From Eq. (5-128), we get the value of C_e

$$C_e = \frac{\Delta I}{V_C \, 8f} = \frac{2}{4 \times 8 \times 10 \, kHz} = 6.25 \, \mu \text{F}$$

Assuming a linear rise of load current i_L during the time from t = 0 to $t_1 = k$ T, Eq. (5-129) gives the approximate value of L as

$$L_{e} = \frac{k T \Delta V_{C}}{\Delta I_{L}} = \frac{k \Delta V_{C}}{\Delta I_{L} f} = \frac{0.7273 \times 4}{0.5 \times 10 \, kHz} = 0.582 \text{ mH}$$

Problem 5-15 PSpice simulation

Problem 5-16

k = 0.4, R = 150 $\Omega,$ r_L = 1 Ω and r_c = 0.2 $\Omega.$

k := 0.5 R := 150 $r_L := 1$ $r_c := 0.2$

(a) Buck

$$G(k) := \frac{k \cdot R}{R + r_L} \qquad \qquad G(0.5) = 0.5$$

(b)Boost

$$G(k) := \frac{1}{1-k} \left[\frac{(1-k)^2 \cdot R}{(1-k)^2 \cdot R + r_L + k \cdot (1-k) \cdot \frac{r_c \cdot R}{r_c + R}} \right] \qquad G(0.5) = 1.95$$

(c)Buck-Boost

$$G(k) := \frac{-k}{1-k} \cdot \left[\frac{(1-k)^2 \cdot R}{(1-k)^2 \cdot R + r_L + k \cdot (1-k) \cdot \frac{r_c \cdot R}{r_c + R}} \right] \qquad G(0.5) = -0.97$$